Autonomous Parallel Parking of a Nonholonomic Vehicle

Igor E. Paromtchik and Christian Laugier
INRIA Rhône-Alpes, GRAVIR
655, avenue de l'Europe, 38330 Montbonnot Saint-Martin, France
Phone: +(33) 76 61 52 92; Fax: +(33) 76 61 52 52; E-mail: Igor.Paromtchik@inrialpes.fr

Abstract

This paper addresses a problem of autonomous parking a nonholonomic (car-like) vehicle. This research work is carried out within the framework of the PRAXITELE project which aims to develop a future-oriented urban transportation system based on a fleet of electric computer-driven vehicles. At the present stage of the project, the designed vehicles have been equiped for the computer-driven motion and their autonomous abilities are being worked out. A practical approach to motion generation and control for autonomous parallel parking of a nonholonomic vehicle is proposed. It is based on range measurements to environmental objects around the vehicle. The motion generation and control is considered within the reactive control scheme to avoid collisions with obstacles. The developed approach is tested on a LIGIER electric autonomous vehicle and is illustrated by the experimental results obtained.

1. Introduction

The autonomous parking problem attracts a great deal of attention from the research community and the automobile industry because of the complexity of this problem for nonholonomic vehicles (cars) and the possibility of numerous practical applications. The motion control for autonomously parking a nonholonomic vehicle (a system with a non-integrable velocity constraint) is within the general problem of steering the vehicle from an arbitrary point to a specified one. Current approaches to solve this general problem can be classified into two main groups: 1 - stabilization of the vehicle to a point by means of a state feedback; 2 - planning a feasible path to reach a point and subsequently following the path. By Brockett's necessary stability conditions [1], a system with non-integrable velocity constraints is open-loop controllable, but it can not be stabilized to a point by means of smooth time-invariant state feedback. To stabilize such a system, time-varying feedback laws are developed by Samson [3], piecewise continuous laws are considered by Canudas de Wit and Sørdalen [4], and discontinuous feedback laws by Guldner and Utkin [5]. Murray and Sastry [6] worked on steering a nonholonomic system between arbitrary points by means of sinusoids. Various path planning approaches are developed by Latombe [2], Laumond *et al.* [7], Kanayama and Hartman [8], van Brussel *et al.* [9] to generate feasible paths for nonholonomic vehicles.

For the "feedback" approaches, accurate localization of the vehicle has to be provided during the motion. The "feedback" and "planning" approaches require accurate kinematic and dynamic models of the vehicle and its environment. However, within these models some of the vehicle's parameters are uncertain (e.g. mass of the vehicle, radius of tires) or unknown (e.g. disturbance forces). An accurate localization of the moving vehicle as well as moving environmental objects remains a significant problem. The "planning" approaches have to result in a feasible reference path for the vehicle. If a planned reference path differs from a feasible one because of unmodelled dynamics or inaccuracies within the models, the vehicle is unable to follow it accurately.

While uncertainces and inaccuracies exist and they are unavoidable, we propose, in order to provide a robust operation for autonomous parking, a combination of "feedback" and "planning". The key idea is to carry out a motion control procedure involving an iteratively repeated *"Localization-Planning-Execution"* cycle until a specified location of the vehicle relative to its environment is reached. To localize the vehicle's coordinates relative to environmental objects, a range measurement system is used. Then, feasible controls which approximately correspond to a path leading to a specified location are planned. These controls are executed by the vehicle's servosystems. After the motion is carried out, the range data processing provides new data for deciding whether the necessary location with respect to environmental objects is reached and the parking maneuver is completed. In order to ensure collision-free motion, the motion control is considered within the reactive control scheme [10].

2. Problem Specification

A four-wheeled electric vehicle with front driven and steering wheels is considered. The vehicle's location relative to some reference coordinate system is denoted as $(x,\, y,\, \theta)$, where x and y are coordinates of the midpoint of the vehicle's rear wheel axle and θ is the orientation angle of the vehicle's frame, as shown in Fig. 1. The motion of

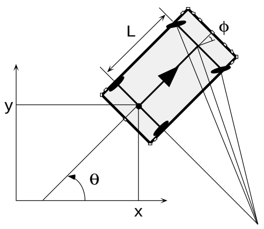

Figure 1: model of a nonholonomic vehicle

the vehicle is described by the following equations:

$$\begin{cases} \dot{x} = v\,\cos\phi\,\cos\theta, \\ \dot{y} = v\,\cos\phi\,\sin\theta, \\ \dot{\theta} = \frac{v}{L}\,\sin\phi, \end{cases} \quad (1)$$

where ϕ is the steering angle, v is the longitudinal velocity of the midpoint of the front wheel axle and L is the wheel base. The vehicle is controlled by the steering angle and the longitudinal velocity, i.e. there are two controls $(\phi,\, v)$, but its location is described by three coordinates $(x,\, y,\, \theta)$ in the plane. Equations (1) correspond to a system with nonholonomic constraints because they involve the derivatives of the vehicle's coordinates and are non-integrable [2]. Equations (1) are valid for a vehicle moving on flat ground where a pure rolling contact without slippage between the wheels and the ground is provided.

The parking structures for vehicles in the urban environment may be classified as "lane", "diagonal" and "row" [11]. In the case of the lane structure the parking bays are oriented parallel to the traffic lane, as in Fig. 2. This structure is mainly used for parking along the streets. The diagonal and row structures are typical for car parks or squares. In this paper the lane structure for parking is considered. Let a vehicle be in a traffic lane within a parking area. The motion control problem for autonomous parking of the vehicle consists of the following subproblems to be solved sequentially: 1 - localization of the parking bay,

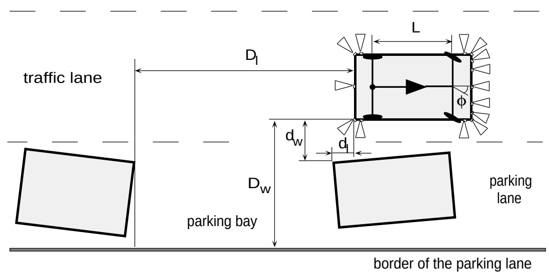

Figure 2: start location for the parking maneuver

2 - adjusting the vehicle relative to the bay to a start location, 3 - parking maneuver (motion into the parking bay).

During localization mode the vehicle moves slowly along the traffic lane. The range data processing allows the building a local map of the environment at the sides of the vehicle. Free spacing is detected and borders of the free bay are localized. The orientation of the bay is calculated and dimensions of the bay are compared with those of the vehicle. The decision on suitability of the bay for parking is made. It is known from experience of driving that, before the parking maneuver starts, the vehicle's position and orientation must be adjusted relative to the location of the parking bay. The vehicle must be oriented near parallel to the parking bay and it must also reach a suitable start position in front of the bay. The parallel parking maneuver results in lateral displacement of the vehicle into the parking bay.

3. Parallel Parking Maneuver

A parallel parking maneuver consists of N iteratively repeated low-speed motions (backwards-forwards) with the coordinated lateral and longitudinal control aimed at obtaining the lateral displacement of the vehicle. The word "parallel" indicates that the start and end orientations of the vehicle are the same as for each iteration $i = 1, \ldots, N$ both for the whole maneuver. The vehicle's orientation varies during the iterative motion. The number N of such motions depends on the longitudinal spacing available within the parking bay and the necessary parking "depth" which depends on the width of the vehicle.

An iterative algorithm for a parallel parking maneuver is developed in [11], where the idea of sinusoidal control functions is applied [6]. For an i-th iterative motion, the vehicle's start coordinates are denoted, omitting the index "i", $x_0 = x(0)$, $y_0 = y(0)$, $\theta_0 = \theta(0)$ and the end coordinates are denoted as $x_T = x(T)$, $y_T = y(T)$, $\theta_T = \theta(T)$, where T is duration of the iterative motion. The following controls are considered:

$$\phi(t) = \phi_{max}\, k_\phi\, A(t), \quad 0 \le t \le T, \qquad (2)$$

$$v(t) = v_{max}\, k_v\, B(t), \quad 0 \le t \le T, \qquad (3)$$

where $\phi_{max} > 0$ and $v_{max} > 0$ are the admissible magnitudes of the steering angle and longitudinal velocity respectively, $k_\phi = \pm 1$ corresponds to the right (+1) or left (–1) parking bay relative to the vehicle's location, $k_v = \pm 1$ corresponds to the forward (+1) or backward (–1) motion, the functions $A(t)$ and $B(t)$ are taken as

$$A(t) = \begin{cases} 1, & 0 \le t < t', \\ \cos \frac{\pi(t-t')}{T^*}, & t' \le t \le T - t', \\ -1, & T - t' < t \le T, \end{cases} \qquad (4)$$

$$B(t) = 0.5\,(1 - \cos \frac{4\pi t}{T}), \quad 0 \le t \le T, \qquad (5)$$

where $t' = \frac{T - T^*}{2}$, $T^* < T$.

A form of the $(x,\, y)$-path corresponding to the controls (2) and (3) is shown in Fig. 3 where, for simplicity, the iterative motion starts from the origin of the reference coordinate system and normalized coordinates are used. One should note

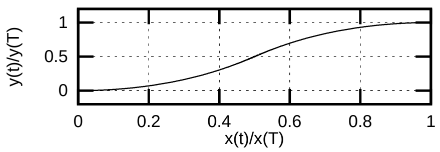

Figure 3: an iterative $(x,\, y)$-path

that the controls (2) and (3) are open-loop in the $(x,\, y,\, \theta)$-coordinates. The vehicle's servosystems provide the planned iterative motion by executing the controls (2) and (3). The resulting accuracy of the motion in the $(x,\, y,\, \theta)$-coordinates depends on the accuracy of the servosystems. Possible errors are compensated by subsequent iterative motions.

For each pair of successive motions $(i,\, i+1)$ the coefficient k_v in (3) has to satisfy the equation $k_{v,i+1} = -k_{v,i}$ that alternates between forward and backward directions. Between successive motions, when the velocity is null, the steering wheels turn to the opposite side in order to obtain the suitable steering angle ϕ_{max} or $-\phi_{max}$ for starting the next iterative motion.

In this way, a form of the controls (2) and (3) is defined by (4) and (5) respectively. In order to evaluate (2)-(5), durations T^*, T and a magnitude ϕ_{max} must be known. The value of T^* depends on the kinematic and dynamic constraints of the steering servosystem [11]. The computation of T and ϕ_{max} is aimed at obtaining their maximal values such that the following "longitudinal" and "lateral" conditions are still satisfied:

$$|\,(x_T - x_0)\, \cos\theta_0 + (y_T - y_0)\, \sin\theta_0\,| < D_l, \quad (6)$$

$$|\,(x_0 - x_T)\, \sin\theta_0 + (y_T - y_0)\, \cos\theta_0\,| < D_w, \quad (7)$$

where $\theta_0 \approx \theta_T$ and a pair $(D_l,\, D_w)$ describes available longitudinal and lateral displacements for the vehicle within the parking bay, as shown in Fig. 2 for the case of the backward motion.

At each iteration i the parallel parking algorithm is summarized as follows:

1. Obtain available longitudinal and lateral displacements $(D_l,\, D_w)$ by processing the range data.
2. Search for maximal values T and ϕ_{max} by evaluating the model (1) with controls (2), (3) so that conditions (6), (7) are still satisfied.
3. Steer the vehicle by controls (2), (3) while processing the range data for collision avoidance.
4. Obtain the vehicle's location relative to environmental objects at the parking bay. If the "parked" location is reached, stop; else, go to step 1.

The start position for the parallel parking maneuver must ensure that subsequent motion will be collision-free with the borders of a parking bay. For a detected parking bay, the range data processing provides distances D_l, D_w, d_l and d_w, as shown in Fig. 2. To be at a suitable start position for further motion into the bay, the vehicle has to stop at a suitable distance d_l computed from distances D_l, D_w, d_w that will ensure a desired minimal safety distance d between the vehicle and the nearest corner of the bay during the motion. For the right side parking in Fig. 2, the left front corner of the bay is the nearest corner that has to be considered. During the backward motion into the bay, as in Fig. 2, the front right corner of the vehicle can run into collision with the front left corner of the bay (see also Fig. 4 where the motion of the vehicle's corners is plotted). In the case of the left side parking, the left front corner of the vehicle and the right front corner of the parking bay must be considered. The relation between the distances D_l, D_w, d_l, d_w and d is described by a function

$$F(D_l,\, D_w,\, d_l,\, d_w,\, d) = 0. \qquad (8)$$

This function can not be expressed in closed form, but it can be estimated for a given type of vehicle by using the model (1) with the controls (2) and (3). The computations are carried out off-line for discrete values of $D_l \in [D_{l,1},\, D_{l,2}]$, $D_w \in [D_{w,1},\, D_{w,2}]$, $d_l \in [d_{l,1},\, d_{l,2}]$ and $d_w \in [d_{w,1},\, d_{w,2}]$; the corresponding distances $d \in [d_1,\, d_2]$ are obtained and stored in a look-up table. This table is used on-line, to obtain an estimate of the longitudinal distance d_l corresponding to a desired minimal safety distance $d \in [d_1,\, d_2]$ and the given D_l, D_w and d_w. An estimate of d_l can be computed as

$$d_l = d_{l,0}(D_l,\, D_w,\, d_w) - d \cdot k(D_l,\, D_w,\, d_w), \quad (9)$$

where $d_{l,0}(D_l,\, D_w,\, d_w)$ corresponds to a value of d_l when $d_1 = 0$, and $k(D_l,\, D_w,\, d_w)$ describes a decline coefficient.

4. Experimental Results

A LIGIER electric autonomous vehicle being developed within the framework of the PRAXITELE project is used for our experiments. This is a four-wheeled vehicle with front-driven and steering wheels. The vehicle has the following dimensions: length $l = 2.5\,m$, wheel base $L = 1.785\,m$ and width $w = 1.4\,m$. The weight of the vehicle is about 600 kg. The vehicle is equiped with a 12 kW asynchronous motor and can move with a maximal velocity of 70 km/h. The steering system is equiped with a DC motor. The vehicle can transport two people. In the case of the parking maneuver, the longitudinal velocity is limited by $v_{max} = 0.75\,m/s$, and the magnitude of the steering angle is $\phi_{max} = 0.4\,rad$.

The control system of the vehicle is based on a Motorola VMEbus system with one VME162-CPU board. The developed steering and velocity control was implemented using ORCCAD software [12] running on a SUN workstation. The compiled code was transmitted via Ethernet to the Motorola VME162 of the vehicle. The steering angle is measured by an optical encoder. Two optical encoders at the rear wheels provide data to compute the longitudinal velocity of the vehicle. The sampling period for the steering and velocity PID-control is 5 ms. The range measurement system in the current configuration consists of 14 ultrasonic sensors (Polaroid 9000) delivering range data with a sampling period of 60 ms. The positions of the sensors on the vehicle's frame are schematically shown in Fig. 2.

An example of the parallel parking maneuver between two vehicles is shown in Fig. 4 for the case of $D_l = 5.5\,m$, $D_w = 2.7\,m$ relative to the start location of the vehicle and the length of the parking bay of 4.6 m. The motion of the vehicle's corners as well as the midpoint of the rear wheel axle is plotted in Fig. 4. In this case, a lateral dis-

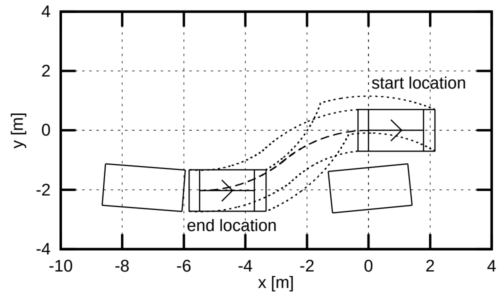

Figure 4: execution of the parking maneuver

tance $d_w = 0.6\,m$ was measured by an ultrasonic range sensor. A longitudinal distance $d_l = 0.9\,m$ was estimated and provided. This has led to "increased" length of the parking bay (5.5 m instead of 4.6 m), and the vehicle has reached the necessary parking "depth" by one iterative motion.

As it is seen from Fig. 4, the front right corner of the vehicle could run into collision with the front left corner of the parking bay, if a longitudinal distance d_l was not properly provided. In Fig. 5 an example of the precomputation of (9) for a LIGIER vehicle is shown. For given lengths (3 m, 4 m and 5 m) and width (2.1 m) of the parking bay, the longitudinal distance d_l is plotted as a function of the minimal safety distance d between the front right corner of the vehicle and the front left corner of the parking bay. One should note that, because of the symmetry, this is also valid for the left side parking when the distance d between the front left corner of the vehicle and the front right corner of the parking bay must be considered. Lateral distances d_w varying from 0.1 m to 1.3 m are also indicated in Fig. 5.

A sequence of motions during an autonomous parking in a dynamic environment is shown in Fig. 6. The parking bay is between the two white vehicles. Step 1: LIGIER was driven to a position near the bay, the driver started the autonomous parking and left the vehicle. Step 2: LIGIER moves forward to localize the bay. Step 3: an obstacle is detected on the way, LIGIER slows down and stops to avoid the collision. Step 4: LIGIER continues its forward motion when the way is free. Step 5: the parking bay is detected and the decision to carry out the parking maneuver is made, LIGIER moves forward to the start position. Step 6: LIGIER stops at the start position. Step 7: while LIGIER moves backwards into

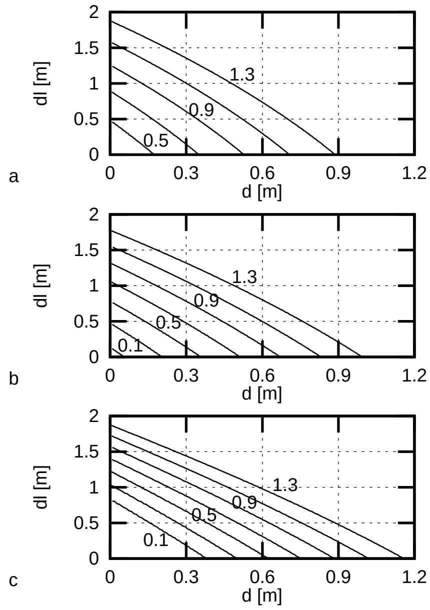

Figure 5: a function $d_l(d)$ for a length of the parking bay: a - $3\,m$, b - $4\,m$, c - $5\,m$

the parking bay, the front human-driven vehicle starts to move backwards, reducing the length of the bay. Step 8: LIGIER continues to move backwards and stops at the rear border of the bay, the range data processing shows that the necessary "depth" is not reached and a further iterative motion has to be carried out. Step 9: LIGIER moves forward to the front border of the bay. Step 10: LIGIER stops at the front border of the bay. Step 11: LIGIER moves backwards and stops at the rear border of the bay. Step 12: the necessary parking "depth" is reached, LIGIER moves forward to the middle between the two vehicles.

5. Conclusion

Motion generation and control for the problem of autonomously parallel parking a nonholonomic vehicle have been considered. An iterative algorithm for the parallel parking maneuver has been developed. This algorithm is based on on-line range readings of the environment around the vehicle. Sinusoidal reference functions for the vehicle's steering and velocity servosystems are used to control the motion during the parallel parking maneuver. The parking control developed was implemented on the LIGIER electric autonomous vehicle. The experimental results obtained have shown the effectiveness of the developed algorithm for autonomously parallel parking a nonholonomic vehicle.

Acknowledgement

This work was funded by the INRIA-INRETS PRAXITELE Program and the ERNET HCM Network of the European Community. We would like to thank the PRAXITELE and SHARP teams for the creative atmosphere during our work.

References

[1] R. W. Brockett, "Asymptotic Stability and Feedback Stabilization," *Differential Geometric Control Theory*, R. W. Brockets, R. S. Millman, and H. J. Sussman editors, Birkhaüser, 1983, pp. 181-191.

[2] J.-C. Latombe, *Robot Motion Planning,* Kluwer Academic Publishers, 1991.

[3] C. Samson, and K. Ait-Abderrahim, "Feedback Control of a Nonholonomic Wheeled Cart in Cartesian Space," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Sacramento, USA, April 9-11, 1991, pp. 1136-1141.

[4] C. Canudas de Wit, and O. J. Sørdalen, "Exponential Stabilization of Mobile Robots with Nonholonomic Constraints," *IEEE Trans. on Automatic Control*, Vol. 11, No. 11, 1992, pp. 1791-1797.

[5] J. Guldner, and V. I. Utkin, "Stabilization of Nonholonomic Mobile Robots Using Lyapunov Functions for Navigation and Sliding Mode Control," *Control Theory and Advanced Technology*, Vol. 10, No. 4, 1994, pp. 635-647.

[6] R. Murray, and S. Sastry, "Steering Nonholonomic Systems Using Sinusoids," *Proc. of the IEEE Int. Conf. on Decision and Control*, Dec., 1990, pp. 2097-2101.

[7] J.-P. Laumond, P. E. Jacobs, M. Taïx, and R. M. Murray, "A Motion Planner for Nonholonomic Mobile Robots", *IEEE Trans. on Robotics and Automation*, Vol. 10, No. 5, 1994, pp. 577-593.

[8] Y. Kanayama, and B. I. Hartman, "Smooth Local Path Planning for Autonomous Vehicles," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Scottsdale, USA, May 14-19, 1989, pp. 1265-1270.

[9] H. van Brussel, J. de Schutter, and K. T. Song, "Hierarchical Control of Free-Navigation AGVs," *Proc. of the Int. Workshop on Information Processing in Autonomous Mobile Robots*, Munich, March 5-8, 1991, pp. 105-119.

[10] J. Borenstein, and Y. Koren, "Obstacle Avoidance with Ultrasonic Sensors," *IEEE J. of Robotics and Automation*, Vol. RA-4, No. 2, 1988, pp. 213-218.

[11] I. E. Paromtchik, and C. Laugier, "Motion Generation and Control for Parking an Autonomous Vehicle," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Minneapolis, USA, April 22-28, 1996, pp. 3117-3122.

[12] D. Simon, B. Espiau, E. Castillo, and K. Kapellos, "Computer-Aided Design of a Generic Robot Controller Handling Reactivity and Real-Time Control Issues," *IEEE Trans. on Control Systems Technology*, Dec., 1993, pp. 213-219.

Figure 6: sequence of motions during autonomous parking