Motion Generation and Control for Parking an Autonomous Vehicle

Igor E. Paromtchik and Christian Laugier
INRIA Rhône-Alpes, LIFIA/IMAG
46, avenue Félix Viallet, 38031 Grenoble Cedex 1, France

Abstract

Practical aspects of motion generation and control for parking a nonholonomic autonomous vehicle are considered. An iterative algorithm for the parallel parking maneuver is proposed. It is based on ultrasonic range data processing. To control the steering angle and longitudinal velocity of the vehicle during the parking maneuver, sinusoidal reference functions are used. To prevent collisions, the maneuver is carried out as a reactive motion. The developed control is experimentally verified for a LIGIER electric autonomous vehicle.

1 Introduction

This paper studies the autonomous parking of car-like (nonholonomic) vehicles. The objective of this task is to extend the autonomous abilities of electric vehicles being developed within the framework of the French PRAXITELE project. Motion generation and control for parking deals with computing reference functions for the steering and velocity servosystems of the vehicle. Traditionally, this is carried out on the basis of a reference path provided by a path planner that takes into account the environmental model as well as the vehicle's dynamics and constraints [1, 2]. The path planning must provide a feasible reference path. Kanayama et al. [3, 4] developed smooth local path planning based on clothoids and cubic spirals. Van Brussel et al. [5] proposed combining motion primitives - lines, circles and clothoids to generate feasible trajectories. Den Boer et al. [6] considered paths specified as series of clothoid curves and a "clothoid-trajectory" controller to steer a car along such paths. Walsh et al. [7] worked on open-loop path planning for robots with nonholonomic constraints.

If the reference path differs from a feasible one because of unmodelled dynamics or inaccuracies within the models, the vehicle is unable to follow it accurately. Bestaoui [8] studied cooperation between the

Figure 1: a LIGIER electric autonomous vehicle

generation of trajectories and their subsequent realization by the servosystems. In the case of nonholonomic vehicles, obtaining feasible paths and their further transformation into reference functions for the servosystems is computationally time consuming [9]. Taking into account the reactive motion abilities of autonomous vehicles, the reference path is computed as a sequence of subgoals corresponding to a coarse path. Most attention is paid to generating the feasible reference functions for the vehicle's servosystems and providing collision-free motion to a goal. This issue is considered within the developed motion generation and control algoritm for parking an autonomous vehicle. The developed algorithm is based on ultrasonic range data processing and sinusoidal reference functions [10] for steering and velocity control.

2 Problem Specification

In this paper a four-wheeled electric vehicle with front driven and steering wheels is considered. The vehicle is shown in Fig. 1. The vehicle is equiped with a sensor system providing range measurements between the vehicle and the environmental objects. Let the vehicle's location, i.e. position and orientation, relative to some reference coordinate system be denoted as $(x,\, y,\, \theta)$, where x and y are coordinates of the midpoint of the vehicle's rear wheel axle and θ is the orientation angle of the vehicle's frame. The motion of the vehicle is described by equations

$$\begin{cases} \dot{x} = v\, cos\, \phi\, cos\, \theta, \\ \dot{y} = v\, cos\, \phi\, sin\, \theta, \\ \dot{\theta} = \frac{v}{L}\, sin\, \phi, \end{cases} \qquad (1)$$

where ϕ is the steering angle, v is the longitudinal velocity of the midpoint of the front wheel axle and L is the wheel base [1, 9]. The vehicle is controlled by the steering angle and the longitudinal velocity, i.e. there are two controls $(\phi,\, v)$, but it has three degrees of freedom $(x,\, y,\, \theta)$ in the plane. Equations (1) correspond to a system with nonholonomic constraints and are non-integrable because they involve the derivatives of the vehicle's coordinates [9].

There may be various parking situations and parking maneuvers. Typically, the parking spacing is structured into bays of a quadrangular form. The parking structures for vehicles may be classified as "lane", "diagonal" and "row", as shown in Fig. 2. In the case of the lane structure the parking bays are oriented parallel to the traffic lane. This structure is mainly used for parking along the streets. The diagonal and row structures are typical for car parks or squares. In this paper the lane structure for parking is considered. The parking task is divided into the following subtasks to be solved sequentially: (1) – localization of the parking bay, (2) – adjusting the vehicle relative to the bay to a start location, (3) – parking maneuver.

During localization mode the vehicle moves slowly along the traffic lane. The range data processing allows the building a local map of the environment at the sides of the vehicle. Free spacing is detected and borders of the free bay are localized. The orientation of the bay is calculated and dimensions of the bay are compared with those of the vehicle. The decision on suitability of the bay for parking is made.

It is known from experience of driving, that before the parking maneuver starts, the vehicle's position and orientation must be adjusted to the location of the parking bay. The vehicle must be oriented near parallel to the parking bay and it must also reach a suitable start position. Further motion is carried out as a parallel parking maneuver. This is a maneuver consisting of N iteratively repeated low-speed motions (forwards-backwards) with the coordinated lateral and longitudinal control aimed at obtaining the lateral displacement of the vehicle. The word "parallel" indicates that the start and end orientations of the vehicle are the same as for each iteration $i = 1, \ldots, N$ both for the whole maneuver. The vehicle's orientation varies during the iterative motion. The number N of such motions depends on the longitudinal spacing available within the parking bay and the necessary parking "depth" which depends on the width of the vehicle.

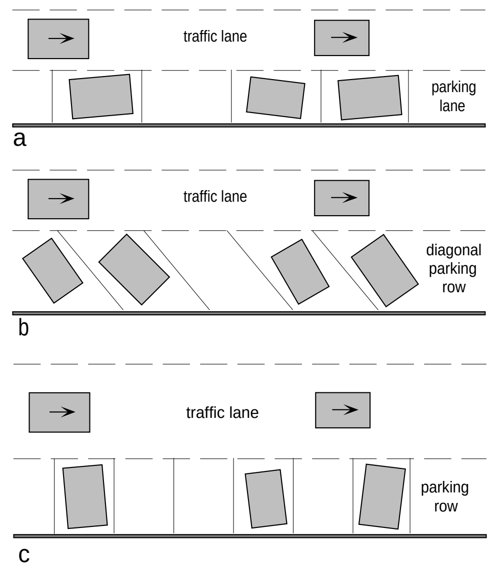

Figure 2: Parking structures: a - lane, b - diagonal, c - row

The motion control for parking is within the general problem of steering a nonholonomic vehicle to a specified location [11, 12]. Since the vehicle is equiped with a range measurement system and location of the vehicle relative to environmental objects is available, this general problem may be simplified in the case of the parking task. Instead of computing a feasible $(x,\, y)$-path leading the vehicle to the specified location, the feasible controls $(\phi,\, v)$ approximately corresponding to a such path are iteratively generated and applied. Between iterative motions, the range data processing provides data for decision making whether the necessary location with respect to environmental objects is reached and the parking maneuver is completed. This avoids problems associated with planning a feasible $(x,\, y)$-path and its subsequent following by the vehicle. In order to ensure collision-free motion, the motion control for parking is considered within the reactive control scheme [13, 14].

3 Parallel Parking Maneuver

Motion control is supported by gathering and processing range data about objects around the vehicle. The range measurements are used for localization of the parking bay, computation of the start location for the parallel parking maneuver, evaluation of the available longitudinal and lateral displacements for the vehicle within the bay, in order to support the collision-free motion and decision making if the parking maneuver is completed. To provide the range measurements, an ultrasonic sensor system is used. The positions of the ultrasonic sensors on the vehicle's frame are schematically shown in Fig. 3. However, the al-

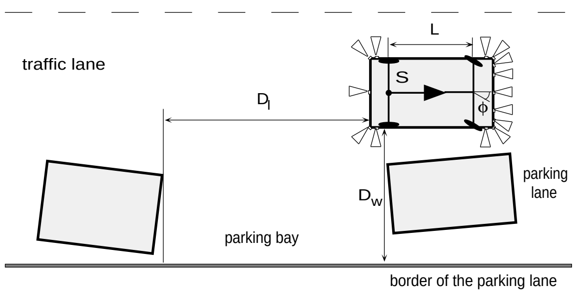

Figure 3: Start location for the parking maneuver

gorithm can easily be extended for the case of more complex sensor systems, e.g. a vision system. Because of the known limitations of the ultrasonic range sensors [13, 15], in order to reduce erroneous measurements, the parking bays may be equiped with a low barrier along the external border of the parking lane so that the "depth" of the parking bay relative to the vehicle's frame can be reliably computed.

Let r_{max} denote the maximal distance being of relevance so that a measurement r_j of a sensor "j" is limited by r_{max}: $r_j \leq r_{max}$. Let r_{min} be the minimal safety distance. Then, weights of the range measurements are

$$w_j = \begin{cases} 0, & r_j \leq r_{min}, \\ \frac{r_j}{r_{max}}, & r_{min} < r_j < r_{max}, \\ 1, & r_j \geq r_{max}. \end{cases} \qquad (2)$$

The weights provide the normalized distances to objects around the vehicle. The minimal weight is defined to be $k_w = \min_j \{w_j\}$ and serves as a multiplying factor when computing the reference velocity, so that the vehicle slows down in the proximity of environmental objects.

During localization, the gathered range data are processed and a local map of the environment for one or both sides of the vehicle is constructed. The detected free space is approximated by a rectangle (D_l, D_w) oriented along the border of the parking bay. The parking is possible, if the dimensions of the bay are large enough in comparison to those of the vehicle. The vehicle adjusts its orientation so as to be parallel to the border of the parking bay. Then, for the obtained orientation θ_s, a start position (x_s, y_s) for the parallel parking maneuver is reached, as it is shown in Fig. 3. The parallel parking maneuver consists of iterative motions. Before each iterative motion starts, the available longitudinal and lateral displacements (D_l, D_w) are recomputed.

For an i-th iterative motion omitting the index "i", the vehicle's start coordinates are denoted $x_0 = x(0)$, $y_0 = y(0)$, $\theta_0 = \theta(0)$ and the end coordinates are denoted as $x_T = x(T)$, $y_T = y(T)$, $\theta_T = \theta(T)$, where T is duration of the iterative motion. The "parallel" condition is defined as

$$\theta_0 - \delta_\theta < \theta_T < \theta_0 + \delta_\theta, \qquad (3)$$

where $\delta_\theta > 0$ is an admissible error in the orientation of the vehicle relative to its start orientation. In fact, this condition allows us to consider the steering control for parking to be similar as that for the lane change maneuver [16].

To control the vehicle during the iterative motion, the idea of sinusoidal functions is applied [10]. The following open-loop controls are considered:

$$\phi(t) = \phi_{max}\, k_\phi\, A(t), \quad 0 \leq t \leq T, \qquad (4)$$

$$v(t) = v_{max}\, k_v\, B(t), \quad 0 \leq t \leq T, \qquad (5)$$

where $\phi_{max} > 0$ and $v_{max} > 0$ are the admissible magnitudes of the steering angle and longitudinal velocity respectively, $k_\phi = \pm 1$ corresponds to the right (+1) or left (–1) parking bay relative to the vehicle's location, $k_v = \pm 1$ corresponds to the forward (+1) or backward (–1) motion, the functions $A(t)$ and $B(t)$ are taken as

$$A(t) = \begin{cases} 1, & 0 \leq t < t', \\ \cos\frac{\pi(t-t')}{T^*}, & t' \leq t \leq T - t', \\ -1, & T - t' < t \leq T, \end{cases} \qquad (6)$$

$$B(t) = 0.5\,(1 - \cos\frac{4\pi t}{T}), \quad 0 \leq t \leq T, \qquad (7)$$

where $t' = \frac{T - T^*}{2}$, $T^* < T$. The (x, y)-path, corresponding to the controls (4) and (5), is shown in Fig. 4, where for simplicity the iterative motion starts from the origin of the reference coordinate system and normalized coordinates are used.

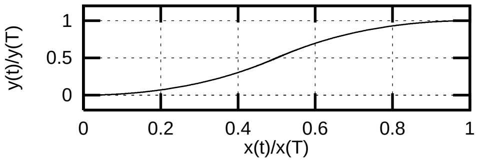

Figure 4: Iterative motion in the $(x,\, y)$-coordinates

For each pair of successive motions $(i,\, i+1)$ the coefficient k_v has to satisfy the equation $k_{v,i+1} = -k_{v,i}$ that alternates between forward and backward directions and vice versa. Between successive motions, when the velocity is null, the steering wheels must turn to the opposite side in order to obtain the suitable steering angle ϕ_{max} or $-\phi_{max}$ for starting the next iterative motion. In this way, a form of the controls (4) and (5) is defined by (6) and (7) respectively. In order to evaluate (4)-(7), durations T^* and T must be computed.

Value of T^* is low-bounded due to the kinematic and dynamic constraints of the steering servosystem, so that for the control (4) one can obtain:

$$T^*_{min} = \pi\, \mathbf{max}\, \{\, \frac{\phi_{max}}{\dot{\phi}_{max}},\ \sqrt{\frac{\phi_{max}}{\ddot{\phi}_{max}}}\, \}, \qquad (8)$$

where $\dot{\phi}_{max}$ and $\ddot{\phi}_{max}$ are the maximal admissible steering rate and acceleration respectively for the steering servosystem. The value of T^*_{min} gives the duration of the full turn of the steering wheels from $-\phi_{max}$ to ϕ_{max} or vice versa, i.e. one can choose $T^* = T^*_{min}$.

Value of T is low-bounded due to the constraints on the velocity v_{max} and acceleration $\dot{v}_{max}$ and due to the condition $T^* < T$, so that for the control (5)

$$T_{min} = \mathbf{max}\, \{\, \frac{2\pi\, v'(D_l)}{\dot{v}_{max}},\ T^*_{min}\, \}, \qquad (9)$$

where the empirically obtained function $v'(D_l) \le v_{max}$ serves to provide the smooth motion of the vehicle.

Computation of T is an iterative procedure based on evaluating the vehicle's model (1) with the controls (4) and (5). This computation is aimed at obtaining the maximal value of T such that the following "longitudinal" and "lateral" conditions are still satisfied:

$$|\, (x_T - x_0)\, \cos\theta_s + (y_T - y_0)\, \sin\theta_s\, | \ <\ D_l, \qquad (10)$$

$$|\, (x_0 - x_T)\, \sin\theta_s + (y_T - y_0)\, \cos\theta_s\, | \ <\ D_w. \qquad (11)$$

The initial estimation is $T_0 = T_{min}$. The maximal value of T is found based on the "longitudinal" condition (10). If condition (10) is satisfied for an estimation T_j, $j = 0, 1, \ldots$, in order to obtain the maximal value of T, the following expression is iteratively used:

$$T_{j+1} = T_j + \Delta T, \quad j = 0, 1, \ldots, \qquad (12)$$

where $\Delta T > 0$. If condition (10) is violated for an estimation T_{j+1}, value of T_j is taken as a duration T of the iterative motion. There may be a situation when for the obtained estimation T_j condition (11) is violated. In this case, in order the "lateral" condition is also satisfied for the estimated value of T, the magnitude of the steering angle ϕ_{max} is iteratively reduced:

$$\phi_{max,k+1} = \phi_{max,k} - \Delta\phi, \quad k = 0, 1, \ldots, \qquad (13)$$

where $\phi_{max,0} = \phi_{max}$ and $\Delta\phi > 0$. The lowest value of the magnitude of the steering angle is given as a constant $\phi_{min} > 0$. The simulation of the iterative motion is repeated for the computed T_j and the new value of the magnitude $\phi_{max,k+1}$: if condition (11) is satisfied, the computational procedure for the iterative motion is finished. The obtained estimations of T and ϕ_{max} are used to evaluate the controls (4) and (5).

One should note, that during the computation of T and ϕ_{max} for the motion from the start location, the motion of the corners of the vehicle's frame is evaluated within the constructed local map of the environment. If the simulation shows a possibility of collisions, the start location of the vehicle is changed. The vehicle moves to a new start location situated farther from the border of the parking lane and nearer to the middle of the parking bay. Then, the computational procedure is repeated.

At each iteration i the parallel parking algorithm is summarized as follows:

1. Obtain available longitudinal and lateral displacements $(D_l,\, D_w)$ by processing the range data.

2. Search for values T and ϕ_{max} by evaluating (1) with controls (4), (5), conditions (10), (11) and using (12), (13).

3. Steer the vehicle by controls (4) and (5) while processing the range data for collision avoidance.

4. Obtain the vehicle's location relative to environmental objects at the parking bay. If the "parked" location is reached, stop; else, go to step 1.

The convergence of the developed algorithm is provided for the computational steps $\Delta T < T_{min}$ and $\Delta\phi < \phi_{min}$. In fact, values T_{min} and ϕ_{min} define the lower bounds $(D^*_l,\, D^*_w)$ for the longitudinal and lateral motions of the vehicle during one iteration. The parking maneuver is possible, if the available longitudinal and lateral spacing $(D_l,\, D_w)$ exceeds these bounds.

4 Experimental Results

A LIGIER electric autonomous vehicle being developed within the framework of the French PRAXITELE project is used for experiments. This is a four-wheeled vehicle with front-driven and steering wheels. The vehicle has the following dimensions: length $l = 2.5\,m$, wheel base $L = 1.765\,m$ and width $w = 1.4\,m$. The weight of the vehicle is about $600\,kg$. The vehicle is equiped with a $12kW$ asynchronous motor and can move with a maximal velocity of $70\,km/h$. The steering system is equiped with a DC motor. The battery resource is about $80\,km$. The vehicle can transport two people. In the case of the parking maneuver, the longitudinal velocity is limited by $v_{max} = 0.3\,m/s$, and the magnitude of the steering angle is $\phi_{max} = 0.4\,rad$.

The control system of the vehicle is based on a Motorola VMEbus system with one VME162-CPU board. The developed steering and velocity control was implemented using ORCCAD software [17] running on a SUN workstation. The compiled code was transmitted via Ethernet to the Motorola VME162 of the vehicle. The steering angle is measured by an optical encoder. Two optical encoders at the rear wheels provide data to compute the longitudinal velocity of the vehicle. A sampling period for the steering and velocity PID-control of $5\,ms$ was obtained. The ultrasonic system in the current configuration consists of 14 range sensors (Polaroid 9000) delivering range data with a sampling period of $60\,ms$.

As it was described in Section 3, the computation of the sufficient T and ϕ_{max} is carried out between the successive iterative motions. Because of the discrete time evaluation, the vehicle's model (1) was rewritten [1], so that for the case $\phi(t_n) = 0$:

$$\begin{cases} \theta(t_n) = \theta(t_{n-1}), \\ x(t_n) = x(t_{n-1}) + v(t_n)\,\Delta t\,\cos\theta(t_n), \\ y(t_n) = y(t_{n-1}) + v(t_n)\,\Delta t\,\sin\theta(t_n), \end{cases} \tag{14}$$

and for the case $\phi(t_n) \neq 0$:

$$\begin{cases} \theta(t_n) = \theta(t_{n-1}) + \frac{v(t_n)\,\Delta t}{L}\,\sin\phi(t_n), \\ x(t_n) = x(t_{n-1}) + \\ \qquad \frac{L}{\tan\phi(t_n)}\,[\sin\theta(t_n) - \sin\theta(t_{n-1})], \\ y(t_n) = y(t_{n-1}) - \\ \qquad \frac{L}{\tan\phi(t_n)}\,[\cos\theta(t_n) - \cos\theta(t_{n-1})], \end{cases} \tag{15}$$

where Δt is a sampling period, $n = 1, 2, \ldots$. When the values of T and ϕ_{max} have been computed, the evaluation of (4) and (5) provides the controls $(\phi,\, v)$ for one iterative motion.

An example of the steering control $\phi(t)$ and its realization by the servosystem is shown in Fig. 5. The steering control was coordinated with the velocity control $v(t)$ shown in Fig. 6 where the computed actual velocity is also plotted. The results obtained show the accurate operation of the steering and velocity servosystems of the LIGIER vehicle.

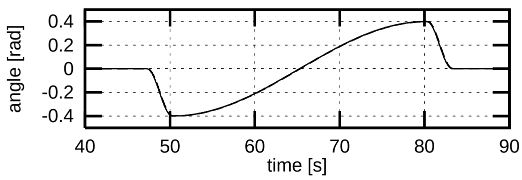

Figure 5: Input and actual steering angles

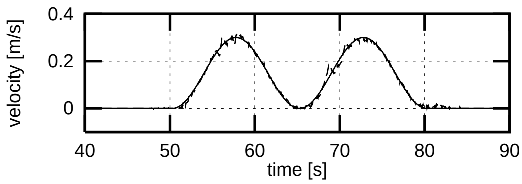

Figure 6: Input and actual velocities

For the steering servosystem the increased errors were examined when the steering wheels turn while the vehicle is not moving (e.g., as in Fig. 5 and Fig. 6, when the wheels turn from 0 to $-\phi_{max}$ and from ϕ_{max} to 0). In this case, the steering servosystem has to overcome the increased forces against the motion of the steering wheels. Because of the dynamical errors of the servosystems and unmodelled effects (e.g., slippage of the wheels or uncertain radius of the tires), the computation of the vehicle's position and orientation according to (14) and (15) leads to an accumulation of error in the estimated vehicle location. In order to reduce these errors and to obtain the vehicle's coordinates more precisely, the vehicle's location is calibrated between iterative motions by processing the range data.

An example of the parallel parking maneuver is shown in Fig. 7 for the case when the available spacing of the parking bay is initially $(4.6\,m,\ 2.1\,m)$ relative to the start location of the vehicle. As it is seen from Fig. 7, three iterative motions are necessary to complete the parking maneuver.

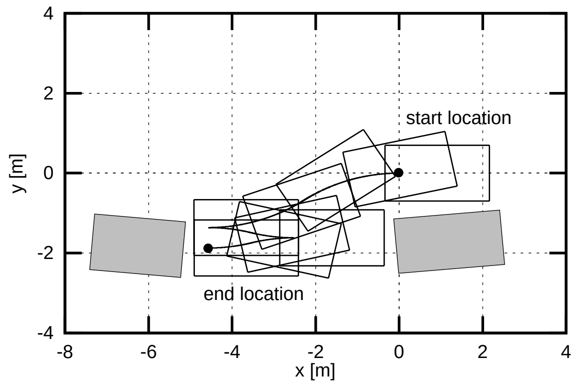

Figure 7: Execution of the parallel parking maneuver

5 Conclusion

Motion generation and control for the task of parking a nonholonomic autonomous vehicle have been considered. An iterative algorithm for the parallel parking maneuver was developed. This algorithm is based on ultrasonic range data processing. The sinusoidal reference functions for the vehicle's steering and velocity servosystems are used to control the motion during the parallel parking maneuver. The developed control for parking was implementated on the LIGIER electric autonomous vehicle. The experimental results obtained have shown the effectiveness of the developed motion generation and control algorithm. Future work will deal with widening the range of the developed control, its further experimental study and extension for various parking maneuvers.

Acknowledgement

This work was funded by INRIA Rhône-Alpes and the ERNET HCM Network of the European Community. The work was carried out at LIFIA/IMAG at Grenoble. Special thanks are given to Dr. M. Parent as well as to S. Abdou, G. Baille, P. Daviet, A. Deguet-Valtrid, Th. Fraichard, P. Garnier, L. Lisowski, C. Novales, D. Pallard – members of the PRAXITELE and SHARP projects for the friendly support and creative atmosphere during our work.

References

[1] J.-C. Latombe, *Robot Motion Planning,* Kluwer Academic Publishers, 1991.

[2] Th. Fraichard, and C. Laugier, "Dynamic Trajectory Planning, Path-velocity Decomposition and Adjacent Paths," *Proc. of the Int. Joint Conf. on Artificial Intelligence*, Vol. 2, Chambéry, France, Sept., 1993, pp. 1592-1597.

[3] Y. Kanayama, and N. Miyake, "Trajectory Generation for Mobile Robots," *Int. J. of Robotics Research*, Vol. 3, 1986, pp. 333-340.

[4] Y. Kanayama, and B. I. Hartman, "Smooth Local Path Planning for Autonomous Vehicles," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Scottsdale, Arizona, May 14-19, 1989, pp. 1265-1270.

[5] H. van Brussel, J. de Schutter, and K. T. Song, "Hierarchical Control of Free-Navigation AGVs," *Proc. of the Int. Workshop on Information Processing in Autonomous Mobile Robots*, Munich, March 5-8, 1991, pp. 105-119.

[6] G. A. den Boer, G. D. van Albada, L. O. Hertzberger, C. Koburg, and M. Mergel, "The MARIE Autonomous Mobile Robot," *Proc. of the Conf. on Intelligent Autonomous Systems IAS-3*, Pittsburgh, Febr. 15-18, 1993, pp. 164-173.

[7] G. Walsh, D. Tilbury, S. Sastry, R. Murray, and J.-P. Laumond, "Stabilization of Trajectories for Systems with Nonholonomic Constraints," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Nice, France, May 12-14, 1992, pp. 1999-2004.

[8] Y. Bestaoui, "Motion Generation with Velocity and Acceleration Constraints," *Robotics and Autonomous Systems*, Vol. 5, No. 3, 1989, pp. 279-288.

[9] J.-P. Laumond, P. E. Jacobs, M. Taïx, and R. M. Murray, "A Motion Planner for Nonholonomic Mobile Robots", *IEEE Trans. on Robotics and Automation*, Vol. 10, No. 5, 1994, pp. 577-593.

[10] R. Murray, and S. Sastry, "Steering Nonholonomic Systems Using Sinusoids," *Proc. of the IEEE Int. Conf. on Decision and Control*, Dec., 1990, pp. 2097-2101.

[11] C. Samson, and K. Ait-Abderrahim, "Feedback Control of a Nonholonomic Wheeled Cart in Cartesian Space," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Sacramento, CA, April 9-11, 1991, pp. 1136-1141.

[12] C. Canudas de Wit, and O. J. Sørdalen, "Exponential Stabilization of Mobile Robots with Nonholonomic Constraints," *IEEE Trans. on Automatic Control*, Vol. 11, No. 11, 1992, pp. 1791-1797.

[13] J. Borenstein, and Y. Koren, "Obstacle Avoidance With Ultrasonic Sensors," *IEEE J. of Robotics and Automation*, Vol. RA-4, No. 2, 1988, pp. 213-218.

[14] I. E. Paromtchik, and U. M. Nassal, "Reactive Motion Control for an Omnidirectional Mobile Robot," *Proc. of the 3rd European Control Conf.*, Roma, Italy, Sept. 5-8, 1995, pp. 3074-3079.

[15] A. M. Flynn, "Combining Sonar and Infrared Sensors for Mobile Robot Navigation," *The Int. J. of Robotics Research*, Vol. 7, No. 6, 1988, pp. 5-14.

[16] C. Hatipoğlu, Ü. Özgüner, and K. A. Ünyelioğlu, "An Optimal Design of a Lane Change Controller," *Proc. of the Symp. Intelligent Vehicles'95*, Detroit, USA, Sept. 25-26, 1995, pp. 436-441.

[17] D. Simon, B. Espiau, E. Castillo, and K. Kapellos, "Computer-Aided Design of a Generic Robot Controller Handling Reactivity and Real-Time Control Issues," *IEEE Trans. on Control Systems Technology*, Dec., 1993, pp. 213-219.