Motion Control for Autonomous Car Maneuvering

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Abstract

This paper focuses on the control methods for autonomous path following and parallel parking of a car-like vehicle. The methods developed are based on a kinematic model of the vehicle. For the path following, a time parameterization of a given path is performed. The proposed control algorithm exponentially stabilizes the motion of the vehicle to the desired feasible path. The autonomous parking is performed as a sequence of controlled motions using sensor data from the car servo-systems and range measurements of the local environment. The methods developed are tested on an experimental automatic car.

1 Introduction

The problem of autonomous car maneuvering attracts a great deal of attention from the research community because of the complexity of this problem for car-like (or nonholonomic) vehicles and the objective of numerous practical applications. A car-like vehicle is a system with a non-integrable velocity constraint [1], [2]. By Brockett's necessary stability conditions [3], such a system is open-loop controllable, but it can not be stabilized to a point by means of smooth time-invariant state feedback. To stabilize such a system, time-varying feedback laws are developed by Samson [4], piece-wise continuous laws are considered by Canudas de Wit and Sørdalen [5], and discontinuous feedback laws by Guldner and Utkin [6]. A tracking controller was proposed by Kanayama *et al.* [7]. Murray and Sastry [8] worked on steering a nonholonomic system between arbitrary points by means of sinusoids. Path planning approaches are developed by Latombe [1] and Laumond *et al.* [2] to generate feasible paths for nonholonomic vehicles.

The kinematics of a nonholonomic vehicle with the front steering wheels is described by the equations [1]

$$\begin{cases} \dot{x} = v\,\cos\phi\,\cos\theta, \\ \dot{y} = v\,\cos\phi\,\sin\theta, \\ \dot{\theta} = \frac{v}{L}\,\sin\phi, \end{cases} \tag{1}$$

where, as shown in Fig. 1, x and y are the Cartesian coordinates of the midpoint of the rear wheel axle, θ is the orientation angle of the vehicle, v is the velocity of the midpoint of the front wheel axle, ϕ is the steering angle, and L is the wheel base. The steering angle and locomotion velocity are the control commands (ϕ, v). Let $\mathbf{z}(t) = (x(t),\, y(t))^T$ and $\mathbf{Z}(t) = (X(t),\, Y(t))^T$ denote vectors of the Cartesian coordinates of the midpoints of the rear and front axles respectively:

$$\mathbf{Z}(t) = \mathbf{z}(t) + L \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}. \tag{2}$$

The velocity of the midpoint of the rear wheel axle is expressed as $v_r = v\,\cos\phi$.

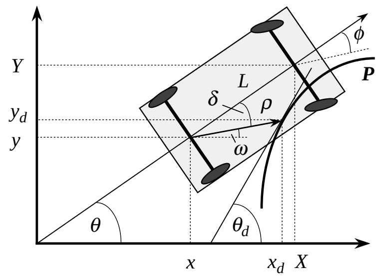

Figure 1: Kinematics of a nonholonomic vehicle

The equations (1) are valid for a vehicle moving on flat ground with a pure rolling contact without slippage between the wheels and the ground. This purely kinematic model is adequate to control low-speed motion of the vehicle. For the high-speed motion, the dynamics of the vehicle must also be considered. In the current implementation, the related velocity and acceleration constraints of the vehicle are only taken into account.

The present paper focuses on the control methods for autonomous path following and parallel parking of a car-like vehicle in a structured traffic environment. This work contributes to the French programmes *Praxitèle* and *La route automatisée* on the development of a new urban transportation system based a fleet of electric vehicles with autonomous motion capabilities.

2 Path Following

Let $\mathbf{P}$ be a curve of a desired path: $\mathbf{P} = \{\, \mathbf{z}_d(s) \mid s \geq 0\}$ where $\mathbf{z}_d(s) = (\, x_d(s), y_d(s)\,)^T$ is a smooth vector function. It is supposed that the parameterization $s(t)$ is nonsingular, $\|\frac{d}{ds}\mathbf{z}_d(s)\| \neq 0$ for all $s \geq 0$ and the first and second derivatives of $\mathbf{z}_d(s)$ are bounded: $\|\frac{d}{ds}\mathbf{z}_d(s)\| \leq C_z^{(1)}$ and $\|\frac{d^2}{ds^2}\mathbf{z}_d(s)\| \leq C_z^{(2)}$.

Let $\theta_d(s)$ denote the angle between the abscissae axis and the tangent vector to the curve $\mathbf{P}$ at a point $\mathbf{z}_d(s)$, as shown in Fig. 1. Because the steering angle is upper-bounded on car-like vehicles, the curvature of $\mathbf{P}$ is assumed to be bounded:

$$|\frac{d}{ds}\theta_d(s)| \leq C_\theta. \tag{3}$$

Let $v_{r,d}(s) \neq 0$ denote a desired velocity of the vehicle.

The control goal is the asymptotic following of a desired path at a desired velocity. It means that it is necessary to provide such controls $(\phi,\, v)$ that conditions

$$\|\mathbf{Z}(t) - \mathbf{z}_d(s(t))\| < \varepsilon_z, \quad \varepsilon_z > 0, \tag{4}$$

$$|v_r(t) - v_{r,d}(s(t))| < \varepsilon_v, \quad \varepsilon_v > 0 \tag{5}$$

hold for arbitrary small ε_z and ε_v for all $t > 0$. In order to focus on the steering control, we assume that (5) is satisfied. The problem is to find a time parameterization $s(t)$ of $\mathbf{P}$ and a steering angle $\phi(t)$ which ensure (4).

Let the distance between the current position $\mathbf{z}(t)$ of the vehicle and the desired position $\mathbf{z}_d(s(t))$ be given by the function $\rho\,[\,\mathbf{z}(t), \mathbf{z}_d(s(t))\,] = \|\,\mathbf{z}_d(s(t)) - \mathbf{z}(t)\,\|$. Let a time parameterization $s(t)$ of $\mathbf{P}$ be chosen in such a way that

$$\rho\,[\,\mathbf{z}(t), \mathbf{z}_d(s(t))\,] = L. \tag{6}$$

Instead of solving this nonlinear algebraic equation online, the following differential equation is used:

$$\frac{d\rho}{dt} + \gamma_\rho(\rho - L) = 0, \quad \gamma_\rho > 0 \tag{7}$$

which ensures that $\rho\,[\,\mathbf{z}(t), \mathbf{z}_d(s(t))\,]$ exponentially converges to L.

Let functions

$$\omega\,[\,\mathbf{z}(t), \mathbf{z}_d(s(t))\,] = \arg\,[\,\mathbf{z}_d(s(t)) - \mathbf{z}(t)\,], \tag{8}$$

$$\delta\,[\,\mathbf{z}(t), \theta(t), \mathbf{z}_d(s(t))\,] = \omega\,[\,\mathbf{z}(t), \mathbf{z}_d(s(t))\,] - \theta(t) \tag{9}$$

represent *direction* and *deviation* angles respectively, as shown in Fig. 1. By substituting the full derivative of $\rho\,[\,\mathbf{z}(t), \mathbf{z}_d(s(t))\,]$ along the trajectory of the system (1) into the reference equation (7), the following differential equation is obtained:

$$\dot{s} = \frac{v_r \cos\delta - \gamma_\rho(\rho - L)}{\|\frac{d}{ds}\mathbf{z}_d(s)\| \cos(\omega - \theta_d)}, \quad s(0) = 0 \tag{10}$$

which determines the desired time parameterization $s(t)$.

The steering control must ensure that the deviation angle δ tends to zero. Taking into account (6), this will guarantee that the position $\mathbf{Z}(t)$ will tend to the position $\mathbf{z}_d(s(t))$ on the desired path. In order the deviation angle δ exponentially converges to zero, the following reference equation is considered:

$$\frac{d\delta}{dt} + \gamma_\delta\, \delta = 0, \quad \gamma_\delta > 0. \tag{11}$$

By substituting the full derivative of the deviation angle δ given by (9) into the equation (11), the following steering control is derived [9]:

$$\phi = \phi\,(\mathbf{z}, \theta, s, v_r) \stackrel{def}{=} \arctan\frac{(\dot{\omega} - \gamma_\delta(\theta - \omega))\,L}{v_r}, \tag{12}$$

where

$$\begin{aligned}\dot{\omega} = \tfrac{1}{\rho}((\tfrac{d}{ds}y_d(s)\dot{s} - v_r\,\sin\theta)\,\cos\omega - \\ (\tfrac{d}{ds}x_d(s)\dot{s} - v_r\,\cos\theta)\,\sin\omega).\end{aligned} \tag{13}$$

Hence, if the initial conditions satisfy the inequalities

$$\begin{cases} 0 < \rho\,[\,\mathbf{z}(0), \mathbf{z}_d(0)\,] \leq L, \\ |\,\theta_d(0) - \omega(\mathbf{z}(0), \mathbf{z}_d(0))\,| < \frac{\pi}{2}, \end{cases} \tag{14}$$

then, for small enough C_θ, the solution of the closed-loop system (1), (10), (12) exists, is defined for all $t > 0$ and

$$\lim_{t\to\infty} \|\mathbf{Z}(t) - \mathbf{z}_d(s(t))\| = 0. \tag{15}$$

3 Parallel Parking

Autonomous parallel parking involves a controlled sequence of motions, in order to localize a sufficient parking place along the road side, obtain a convenient start location for the vehicle beside the parking place, and perform a parallel parking maneuver. A start location for parallel parking is shown in Fig. 2 where an autonomous vehicle A1 is in a traffic lane. The parking place is between the parked vehicles B1 and B2. L1 and L2 are respectively the length and width of A1. D1 and D2 are the distances available for longitudinal and lateral displacements of A1 within the parking place. D3 and D4 are the longitudinal and lateral displacements of the corner A13 of A1 relative to the corner B24 of B2. If (D1-D3) > L1 and (D2-D4) > L2, the place is sufficiently large for parking the vehicle A1.

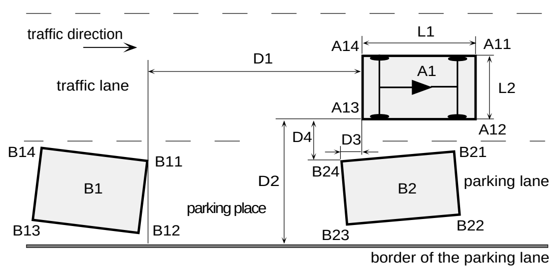

Figure 2: Start location for parallel parking

For the parallel parking maneuver, iterative low-speed backwards-and-forwards motions with coordinated control of the steering angle and locomotion velocity are performed to produce a lateral displacement of the vehicle into the parking place. The number of such motions depends on the distances D1, D2, D3, D4 and the necessary parking "depth" which depends on the width L2 of the vehicle A1. The start and end orientations of the vehicle are the same for each iterative motion $i = 1, \ldots, N$.

For the i-th iterative motion (but omitting the index "i"), let the start coordinates of the vehicle be $x_0 = x(0)$, $y_0 = y(0)$, $\theta_0 = \theta(0)$ and the end coordinates be $x_T = x(T)$, $y_T = y(T)$, $\theta_T = \theta(T)$, where T is duration of the motion. The "parallel parking" condition means that $|\,\theta_T - \theta_0\,| < \delta_\theta$ where $\delta_\theta > 0$ is a small admissible error in orientation of the vehicle.

At each iteration i the following commands of the steering angle and locomotion velocity are applied [10]:

$$\phi(t) = \phi_{max}\, k_\phi\, A(t), \quad 0 \le t \le T, \tag{16}$$

$$v(t) = v_{max}\, k_v\, B(t), \quad 0 \le t \le T, \tag{17}$$

where $\phi_{max} > 0$ and $v_{max} > 0$ are the admissible magnitudes of the steering angle and locomotion velocity respectively, $k_\phi = \pm 1$ corresponds to a right side (+1) or left side (–1) parking place relative to the traffic lane, $k_v = \pm 1$ corresponds to forward (+1) or backward (–1) motion,

$$A(t) = \begin{cases} 1, & 0 \le t < t', \\ \cos \frac{\pi(t-t')}{T^*}, & t' \le t \le T - t', \\ -1, & T - t' < t \le T, \end{cases} \tag{18}$$

$$B(t) = 0.5\left(1 - \cos\frac{4\pi t}{T}\right), \quad 0 \le t \le T, \tag{19}$$

where $t' = \frac{T - T^*}{2}$, $T^* < T$. The $(x,\, y)$-path corresponding to the commands (16) and (17) is shown in Fig. 3, where for simplicity, the iterative motion starts from the origin of the reference coordinate system and normalized coordinates are used.

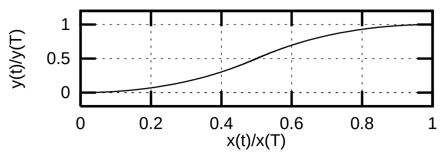

Figure 3: Iterative motion in the $(x,\, y)$-coordinates

The commands (16) and (17) are open-loop in the $(x,\, y,\, \theta)$-coordinates. The steering wheel servo-system and locomotion servo-system must execute the commands (16) and (17), in order to provide the desired $(x,\, y)$-path and orientation θ of the vehicle. The resulting accuracy of the motion in the $(x,\, y,\, \theta)$-coordinates depends on the accuracy of these servo-systems. Possible errors are compensated by subsequent motions.

For each pair of successive motions $(i,\, i+1)$, the coefficient k_v in (17) has to satisfy the equation $k_{v,i+1} = -k_{v,i}$ that alternates between forward and backward directions. Between successive motions, when the velocity is null, the steering wheels turn to the opposite side in order to obtain a suitable steering angle ϕ_{max} or $-\phi_{max}$ to start the next iterative motion.

In this way, the form of the commands (16) and (17) is defined by (18) and (19) respectively. In order to evaluate (16)-(19) for the parallel parking maneuver, the durations T^* and T, the magnitudes ϕ_{max} and v_{max} must be known.

The value of T^* is lower-bounded by the kinematic and dynamic constraints of the steering wheel servo-system. When the control command (16) is applied, the lower bound of T^* is

$$T^*_{min} = \pi\, \mathbf{max} \left\{ \frac{\phi_{max}}{\dot\phi_{max}},\ \sqrt{\frac{\phi_{max}}{\ddot\phi_{max}}} \right\}, \tag{20}$$

where $\dot\phi_{max}$ and $\ddot\phi_{max}$ are the maximal admissible steering rate and acceleration respectively. The value of T^*_{min} gives duration of the full turn of the steering wheels from $-\phi_{max}$ to ϕ_{max} or vice versa, i.e. one can choose $T^* = T^*_{min}$.

The value of T is lower-bounded by the constraints on the velocity v_{max} and acceleration $\dot v_{max}$ and by the condition $T^* < T$. When the control command (17) is applied, the lower bound of T is

$$T_{min} = \mathbf{max} \left\{ \frac{2\pi\, v'(\mathrm{D1})}{\dot v_{max}},\ T^* \right\}, \tag{21}$$

where the empirically-obtained function $v'(\mathrm{D1}) \le v_{max}$ serves to provide a smooth motion when the available distance D1 is small.

The computation of T and ϕ_{max} aims to obtain the maximal values such that the following "longitudinal" and "lateral" conditions are still satisfied:

$$|\,(x_T - x_0)\cos\theta_0 + (y_T - y_0)\sin\theta_0\,| < \mathrm{D1}, \tag{22}$$

$$|\,(x_0 - x_T)\sin\theta_0 + (y_T - y_0)\cos\theta_0\,| < \mathrm{D2}. \tag{23}$$

Using the maximal values of T and ϕ_{max} assures that the longitudinal and lateral displacement of the vehicle is maximal within the available free parking space. The computation is carried out on the basis of the model (1) when the commands (16) and (17) are applied. In this computation, the value of v_{max} must correspond to a safety requirement for parking maneuvers (e.g. $v_{max} = 0.75\ m/s$ was found empirically).

At each iteration i the parallel parking algorithm is summarized as follows:

1. Obtain available longitudinal and lateral displacements D1 and D2 respectively by processing the sensor data.

2. Search for maximal values T and ϕ_{max} by evaluating the model (1) with controls (16), (17) so that conditions (22), (23) are still satisfied.

3. Steer the vehicle by controls (16), (17) while processing the range data for collision avoidance.

4. Obtain the vehicle's location relative to environmental objects at the parking place. If the "parked" location is reached, stop; else, go to step 1.

When the vehicle A1 moves backwards into the parking place from the start location shown in Fig. 2, the corner A12 must not collide with the corner B24. The start location must ensure that the subsequent backward motion will be collision-free with objects limiting the parking place. To obtain a convenient start location, the vehicle has to stop at a such distance D3 that will ensure a desired minimal safety distance D5 between the vehicle A1 and the nearest corner of the parking place during the subsequent backward motion. The relation between the distances D1, D2, D3, D4 and D5 is described by a function

$$\mathcal{F}(\mathrm{D1},\ \mathrm{D2},\ \mathrm{D3},\ \mathrm{D4},\ \mathrm{D5}) = 0. \qquad (24)$$

This function can not be expressed in closed form, but it can be estimated for a given type of vehicle by using the model (1) when the commands (16) and (17) are applied. The computations are carried out off-line and stored in a look-up table which is used on-line, to obtain an estimate of D3 corresponding to a desired minimal safety distance D5 for given D1, D2 and D4 [11].

4 Experiments

The developed methods are implemented on an experimental automatic vehicle designed on the base of a LIGIER electric car, shown in Fig. 4. The notion *automatic vehicle* is used to indicate that the car is equiped with: (1) - a *sensor unit* to measure relative distances between the vehicle and environmental objects, (2) - a *servo unit* for low-level control of the steering angle and locomotion velocity, (3) - a *control unit* that processes data from the sensor and servo units and "drives" the vehicle by issuing appropriate servo commands.

Figure 4: a LIGIER electric car

The control unit is based on a Motorola VME162-CPU board and a transputer net. A VxWorks real-time operating system is used. The sensor unit involves the ultrasonic range sensors (Polaroid 9000) and a linear CCD-camera. The steering wheel servo-system is equiped with a direct current motor and an optical encoder to measure the steering angle. The locomotion servo-system is equiped with 12 kW asynchronous motor and two optical encoders at the rear wheels to provide odometry data. The vehicle also has an hydraulic braking servo-system. The developed control methods are implemented using ORCCAD software [12] running on a SUN workstation. The compiled code is transmitted via Ethernet to the control unit of the vehicle.

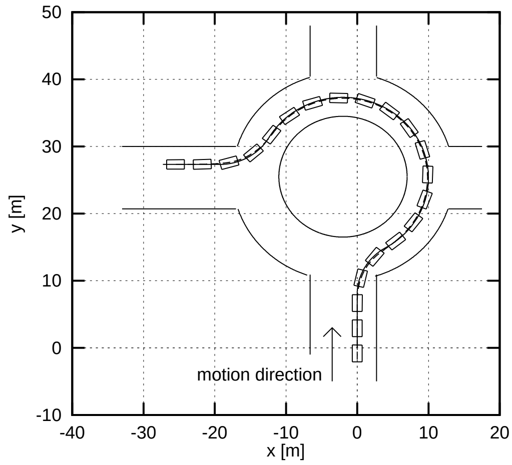

Figure 5: Path following on a circular road

To illustrate the path following, an example of a circular road is shown in Fig. 5. The vehicle follows the desired path given as $(x,\ y)$-points. The locomotion velocity of the vehicle is 1 m/s. The time parameterization $s(t)$ of the path is obtained according to the equation (10). The control command of the steering angle $\phi(t)$ is computed according to the equation (12).

An example of our experimental setup for autonomous parallel parking in a street is shown in Fig. 6. The parking place is in front of LIGIER at its right side between the two vehicles. Autonomous parking can be performed in an environment where there are moving obstacles [13].

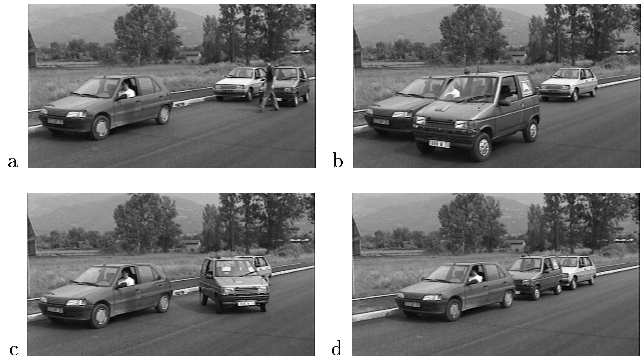

Figure 6: Sequence of motions for parallel parking: a - autonomous motion to localize a parking place, b - obtaining a convenient start location, c - backward motion into the parking place, d - parking is completed

An example of the commands (16) and (17) for parallel parking into a parking place situated at the right side of the vehicle is shown in Fig. 7. The parallel parking maneuver is depicted in Fig. 8 where the motion of the corners of the vehicle and the midpoint of the rear wheel axle is plotted. The dimensions of the vehicle are: L1=2.5 m, L2=1.4 m and L=1.785 m. The available distances are D1=4.9 m and D2=2.7 m relative to the start location of the vehicle. The distance D4=0.6 m was measured by the sensor unit. The distance D3=0.8 m was estimated so as to ensure the minimal safety distance D5=0.2 m. In this case, five iterative motions are performed to park the vehicle. As seen in Fig. 7 and Fig. 8, the durations T of the iterative motions, magnitudes of the steering angle ϕ_{max} and locomotion velocity v_{max} correspond to the available distances D1 and D2 within the parking place (e.g. the values of T, ϕ_{max} and v_{max} differ for the first and last iterative motion).

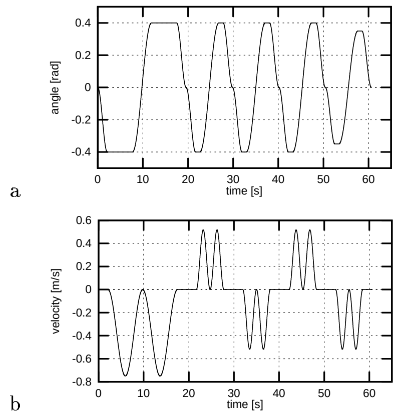

Figure 7: Control commands for parallel parking when backward and forward motions are performed: a - steering angle, b - locomotion velocity

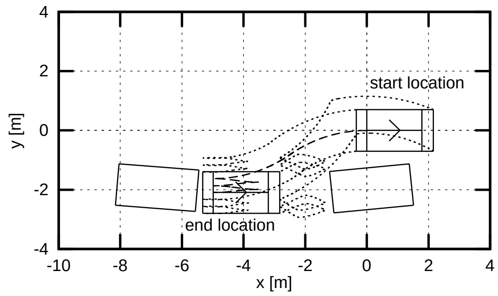

Figure 8: Parallel parking when backward and forward motions are performed

5 Conclusion

Methods to control the motion of a car-like vehicle for autonomous path following and parallel parking were developed. The autonomous maneuvers were considered in a structured traffic environment. The vehicle's kinematic and dynamic constraints were taken into account to ensure the feasibility of the control commands. The methods developed were implemented and tested on an automatic electric vehicle. The experimental results obtained show the effectiveness of the developed methods.

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