Navigating Between People: a Stochastic Optimization Approach

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***Abstract*— The objective of this paper is to present a strategy to safely move a robot in an unknown and complex environment where people are moving and interacting. The robot, by using only its sensor data, must navigate respecting humans' comfort. To obtain good results in such a dynamic environment, a prediction on humans' movement is also crucial. To solve all the aforementioned problems we introduce a suitable cost function. Its optimization is obtained by using a new stochastic and adaptive optimization algorithm (CAO). This method is very useful in particular when the analytical expression of the optimization function is unknown but numerical values are available for any state configuration. Additionally, the proposed method can easily incorporate any dynamical and environmental constraints. To validate the performance of the proposed solution, several simulation results are provided.**

I. INTRODUCTION

Robots navigating close to humans or involved in interaction tasks with humans must assure not only safe but understandable behavior in order to prevent discomfort in people. Recently, several possible solutions to this problem have been proposed [1], [2], [3], [4]. Our work is placed in this framework: we are interested in safely lead a robot in an unknown and complex environment, where people are moving and interacting, respecting the humans' comfort. The first step is to understand how humans manage the space around them while navigating and how their decisions affect the comfort of others. Many psychological theories have been proposed to explain the relation between distance, visual behaviors and comfort in humans (see [5] and references therein). Intuitively people will become uncomfortable if they are approached at a distance that is judged to be too close: the greater invasion/intrusion the more discomfort or arousal is experienced by the person. This simple idea was formalized introducing the concept of personal space, firstly proposed by Hall [6], which characterizes the space around a human being in terms of comfort to social activity. In casual conversations, people claim an amount of space related to that activity. This space is respected by other people and only participants have permitted access to it, therefore the intrusion of a stranger causes discomfort [7]. It can be assumed that people will engage in proxemic behavior with robots in much the same way that they interact with other people [8]. For example in [9], participants evaluated the direct frontal approach as least comfortable for a bring object task by finding robots motion threatening and aggressive.

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The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement n. 231855 (sFly) and from CONACYT 250140/308006.

In this paper we are formulating the problem of social robot navigation as an optimization problem where the objective function includes, in addition to the distance to goal, information about comfort of present humans. We use a new stochastic and adaptive optimization method: the CAO algorithm. Using this method we can also obtain an indirect prediction on the people movement, which is a very crucial point to get good results for a similar task.

The CAO methodology, which was recently introduced in [10], [11], is able to efficiently handle optimization problems for which an analytical form of the function to be optimized is unknown, but the function is available for measurements at each iteration of the algorithm employed to optimize it. As a result, it perfectly suits the optimization problem presented in this paper. Similar situations are very common in robotics and the power of the CAO algorithm to handle such a problem is already shown in [12], [13] for multi-robot cooperative coverage. The CAO approach extends the popular Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm [14]. The difference between the SPSA and the CAO approach is that SPSA employs an approximation of the gradient of an appropriate cost function using only the most recent experiments, while the CAO approach employs linear-in-the-parameters approximators that incorporate information of a user specified time window of the past experiments together with the concept of stochastic candidate perturbations for efficiently optimizing the unknown function.

It is finally mentioned that the CAO or the SPSA do not create an explicit approximation or estimation of obstacles location, humans' movement prediction and other unknown information; instead, they on-line produce a local approximation only of the unknown cost function to optimize. For this reason, they require simple, and thus scalable, approximation schemes to be employed.

A. Related Work

A proposal of human aware navigation was presented in [1], where a motion planner takes explicitly into account its human partners. The authors introduced the criterion of visibility, which is simply based on the idea that the comfort increases when the robot is in the field of view of a person. Our assumption is in some way the opposite of the last criterion: the field of view shows the point of interest of a person then, if the robot enters it, the activity of the person will be interrupted decreasing the comfort function. The work presented in [3] proposed rules that a single robot should obey in order to achieve not only a safe but also a least disturbance motion in a human-robot environment. It is considered the fact that both humans and robots have their sensitive zones, depending either on their security regions or on psychological feeling of humans. Personal space, o-space and their relation to comfort were addressed in [4], where a risk based navigation was extended to include risk due to discomfort. Human's movement is supposed to be known by learning of typical trajectories in a particular environment. In [2] a generalized framework for representing social conventions as components of a constrained optimization problem was presented and it was used for path planning and navigation. Social conventions were modeled as costs for the A* planner with constraints like shortest distance, personal space and pass on the right. In contrast with the previous works, we can take advantage of information about past people positions to obtain indirectly a humans' movement prediction. This fundamental advantage is based on the possibility to work with an unknown objective function.

The rest of the paper is organized as follows: before to present in details the particular problem we want to solve, in next section we explain the main concepts of the adopted method and its mathematical properties; section III formulates the problem approached in this paper and it shows how the proposed optimization method can be applied to find a solution. The performance of the proposed approach is presented in section IV, where several simulation results are shown and the experimental platform where the algorithm will be tested is described. Finally, in section V, the conclusions of this work and possible future extensions are drawn.

II. The CAO Approach

The Cognitive-based Adaptive Optimization (CAO) approach, recently proposed [10], [11], is a new stochastic optimization algorithm very useful if the analytical expression of the function to optimize is unknown. Let us suppose to have an optimization function depending on a set of variables $x_k^{(1)}, \ldots, x_k^{(M)}$ (e.g., the robot's positions):

$$J_k = \mathcal{J}\left(x_k^{(1)}, \ldots, x_k^{(M)}\right) \tag{1}$$

where $k = 0, 1, 2, \ldots$ denotes the time-index, M the state's dimension, J_k the numerical value of the optimization function at the k-th time-step and $\mathcal{J}$ is a nonlinear function which depends, apart from the explicit variables, on the particular environment where the robot lives.

Due to a lack of information, like for example particular environment characteristics, the explicit form of the function $\mathcal{J}$ is not known in most practical situations; as a result, standard optimization algorithms (e.g. steepest descent) are not applicable to the problem in hand. However, in most practical cases the current value can be estimated, e.g. from the robot's sensor measurements. In other words, at each time-step k, an estimate of J_k is available through sensor measurements,

$$J_k^n = \mathcal{J}\left(x_k^{(1)}, \ldots, x_k^{(M)}\right) + \xi_k \tag{2}$$

where J_k^n denotes the estimate of J_k and ξ_k denotes the noise introduced in the estimation of J_k due to the presence of noise in the robot's sensors.

Apart from the problem of dealing with a criterion for which only a sensor-based information is available, an efficient algorithm for real applications has additionally to deal with the problem of restricting the state variables so that obstacle avoidance as well as dynamical constraints are met. In other words, at each time-instant k, the vectors $x_k^{(i)}, i = 1, \ldots, M$ should satisfy a set of constraints which, in general, can be represented as follows:

$$\mathcal{C}\left(x_k^{(1)}, \ldots, x_k^{(M)}\right) \leq 0 \tag{3}$$

where $\mathcal{C}$ is a set of nonlinear functions of the state variables. As in the case of $\mathcal{J}$, the function $\mathcal{C}$ depends on the particular environment characteristics (e.g. location of obstacles, terrain morphology) and an explicit form may be not known in many practical situations; however, it is natural to assume that during the task is possible to get information whether a particular selection of state variables satisfies or violates the set of constraints (3).

Hence, the optimization problem can be described as the problem of changing $x_k^{(1)}, \ldots, x_k^{(M)}$ to a set that solves the following constrained optimization problem: maximize (1) subject to (3). As already noticed, the difficulty in solving in real-time and in real-life situations this constrained optimization problem lies in the fact that explicit forms for the functions $\mathcal{J}$ and $\mathcal{C}$ are not available.

As a first step, the CAO approach makes use of function approximators for the estimation of the unknown objective function $\mathcal{J}$ at each time-instant k according to

$$\hat{J}_k\left(x_k^{(1)}, \ldots, x_k^{(M)}\right) = \vartheta_k^\tau \phi\left(x_k^{(1)}, \ldots, x_k^{(M)}\right). \tag{4}$$

Here $\hat{J}_k$ denotes the approximation/estimation of $\mathcal{J}$ generated at the k-th time-step, ϕ denotes the nonlinear vector of L regressor terms, ϑ_k denotes the vector of parameter estimates calculated at the k-th time-instant and L is a positive user-defined integer denoting the size of the function approximator (4). The parameter estimation vector ϑ_k is calculated according to

$$\vartheta_k = \underset{\vartheta}{\operatorname{argmin}} \frac{1}{2} \sum_{\ell=\ell_k}^{k-1} \left(J_\ell^n - \vartheta^\tau \phi\left(x_\ell^{(1)}, \ldots, x_\ell^{(M)}\right)\right)^2 \tag{5}$$

where $\ell_k = \max\{0, k-L-T_h\}$ with T_h being a user-defined nonnegative integer. Standard least-squares optimization algorithms can be used for the solution of (5).

Remark 1: In order for the proposed methodology to guarantee with efficient performance, special attention has to be paid in the selection of the regressor vector ϕ. The particular choice adopted in this paper is described in section III. ◇

As soon as the estimator $\hat{J}_k$ is constructed according to (4), (5), the set of new state variables is selected as follows: firstly, a set of N candidate state variables is constructed according to

$$x_k^{i,j} = x_k^{(i)} + \alpha_k \zeta_k^{i,j}, i \in \{1, \ldots, M\}, j \in \{1, \ldots, N\}\,, \quad (6)$$

where $\zeta_k^{i,j}$ is a zero-mean, unity-variance random vector with dimension equal to the dimension of $x_k^{(i)}$ and α_k is a positive real sequence which satisfies the conditions:

$$\lim_{k\to\infty} \alpha_k = 0, \quad \sum_{k=1}^{\infty} \alpha_k = \infty, \quad \sum_{k=1}^{\infty} \alpha_k^2 < \infty\,. \qquad (7)$$

Among all N candidate new variables $x_k^{1,j}, \ldots, x_k^{M,j}$, the ones that correspond to non-feasible variables, i.e. the ones that violate the constraints (3), are neglected and then the new state is calculated as follows:

$$\left[x_{k+1}^{(1)}, \ldots, x_{k+1}^{(M)}\right] = \underset{\substack{j \in \{1,\ldots,N\} \\ x_k^{i,j} \text{not neglected}}}{\operatorname{argmin}} \hat{J}_k\left(x_k^{1,j}, \ldots, x_k^{M,j}\right)$$

The idea behind the above logic is simple: at each time-instant a set of many candidate new state variables is stochastically generated and the candidate, among the ones that provide with a feasible solution, that provides the "best" estimated value $\hat{J}_k$ of the optimization function is selected as the new set of state variables. The random choice for the candidates is essential and crucial for the efficiency of the algorithm, as such a choice guarantees that $\hat{J}_k$ is a reliable and accurate estimate for the unknown function $\mathcal{J}$; see [10], [11] for more details. The next theorem summarizes the properties of the CAO algorithm described above:

Theorem 1: Let $x^{(1^*)}, \ldots, x^{(M^*)}$ denote any local minimum of the constrained optimization problem. Assume also that the functions $\mathcal{J}, \mathcal{C}$ are either continuous or discontinuous with a finite number of discontinuities. Then, the CAO algorithm as described above guarantees that the state $x_k^{(1)}, \ldots, x_k^{(M)}$ will converge to one of the local minima $x^{(1^*)}, \ldots, x^{(M^*)}$ with probability 1, provided that the size L of the regressor vector ϕ is larger than a lower bound $\bar{L}$.

The proof of this theorem, not presented here for brevity purposes, is among the same lines as the main results of [10], [11]; the main difference is that while in that case it is established that the CAO algorithm is approximately a *gradient-descent* algorithm, the CAO algorithm used in this paper is proven to be approximately a *projected gradient-descent* algorithm.

Remark 2: As already noticed in section I, the CAO algorithm requires only a local approximation of the unknown function $\mathcal{J}$ and as a result the lower bound $\bar{L}$ has not to be large (as opposed to methods that construct a global approximation of the function $\mathcal{J}$). Although, there exist no theoretical results for providing the lower bound $\bar{L}$ for the size of the regressor vector ϕ, practical investigations on many different problems (even in cases where the dimension of the variables to be optimized is as high as 500; see [10]-[11] for more details) indicate that for the choice of the regressor vectors according to Remark 1 such a bound is around 20. $\diamond$

III. Proposed Solution

In this section we formulate the problem of social robot navigation and we show how the proposed optimization algorithm can be applied in practice to the problem studied in this paper. Furthermore, we discuss how it is possible to include a prediction of humans' motion by using the CAO algorithm.

Our intent is to safely move a robot in a complex and unknown environment respecting the comfort of the people moving in. Let $x_0^{(R)}$ be the robot start position and let $x^{(G)}$ be the goal position. Our intent is to move the robot from $x_0^{(R)}$ to $x^{(G)}$ minimizing the discomfort of humans located at positions $\{p^{(i)}\}$. The discomfort function has two components, one for the invasion of Personal Space ($dis(PS)$) and the other for invasion of Information Process Space ($dis(IPS)$), both of them explained later in this section. To fulfill both the tasks of reaching the goal and respecting the people, we define the optimization function in the following way:

$$J = \lambda * (dis(PS) + dis(IPS)) + D(x^{(G)}) \qquad (8)$$

where λ is a constant parameter and $D(x^{(G)})$ is a function depending on the distance to the goal. In our case it is the Euclidean distance.

The difference with respect to the general presentation of the algorithm, provided in section II, is that now the cost function depends on both active variables (the robot's position $x^{(R)}$) and passive variables (humans' positions $\{p^{(i)}\}$). This means that now the cost function can be expressed in the form:

$$J = J(x^{(R)}; \{p^{(i)}\}) \qquad (9)$$

and only the controllable components $x^{(R)}$ are perturbed to generate the candidate new positions.

A. Discomfort model

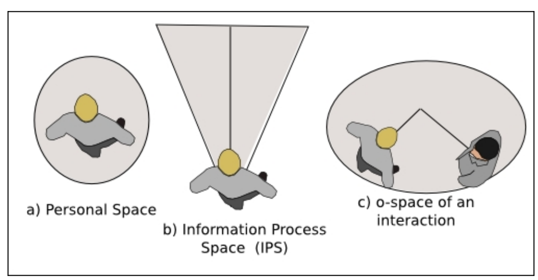


Fig. 1. We consider as discomfort the invasion made to humans' space by the robot, specifically, a) Personal Space b) Information Process Space or c) o-space.

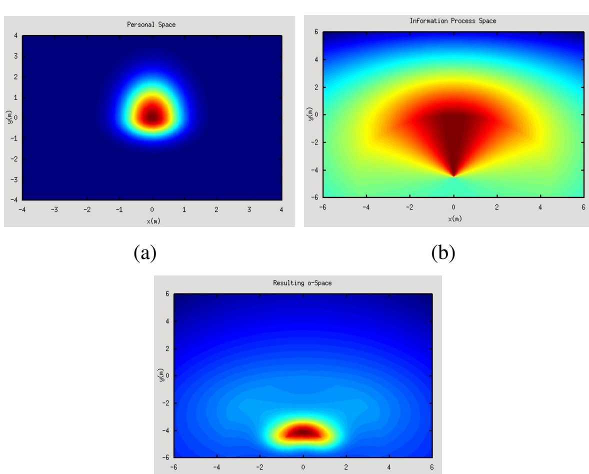
Since comfort is a subjective notion it is clear that it cannot be measured directly by any sensor, however studies have been developed to explain how distance and visual behavior affect comfort in humans (see [5] for a review). Some other works have studied the visual behavior of pedestrians when navigating: for example in [15] authors explored the size and the shape of Information Process Space (IPS), in which a pedestrian takes into account other pedestrians and obstacles for calculating next moves and where psychological comfort is evaluated (this space can be related to visual field). Inspired by these works, our model considers as discomfort the invasion made to humans' space, specifically personal space [6], o-space [7] and Information Process Space [15], by the robot. A representation of these spaces can be observed in Fig. 1. We assume that the discomfort will be higher in the spaces previously mentioned and we propose a function that approximates them. The function to represent IPS is inspired on the representation of the Doppler effect which establishes that the perception in the frequency of a sound varies with the movement of source and observer. The source of sound is a pedestrian that moves with a constant velocity and all the other points are observers which do not move. Then the equation is:

$$f' = \frac{c}{c - v_s \cos\theta_s} f \,, \tag{10}$$

where f is the frequency emitted by the source, f' is the frequency perceived by the observer, c is the velocity of sound, v_s is the velocity of the source and θ_s is the angle between the direction of the source and the direction of the line that links observer and source. The numerical values of the parameters in eq. (10) have been determined empirically to best adjust the results for IPS in [15]. They are $c = 3.43$, $v_s = 3.0$ and f is determined in function of distance as stated in next equation:

$$f = \begin{cases} 1 & if \quad d < d_e \\ 1 - \left(\frac{d-d_e}{d_l}\right) & if \quad d_e \leq d \leq d_e + d_l \\ 0 & if \quad d > d_e + d_l \end{cases} \tag{11}$$

where d is the distance from the human's position, d_e is the main radius of IPS effect and d_l is the range where the IPS loses its effect. In our current implementation $d_e = 4.5$ and $d_l = 4.5$.



(a) (b)

(c)

Fig. 2. Models implemented to represent discomfort in humans' spaces: (a) Personal Space of a human in (0,0) and orientation of 90 degrees. (b) IPS for a human in (0,-4.5) and orientation of 90 degrees. (c) O-space for two humans in positions (-0.85,-4.5) and (0.85,-4.5) and orientations of 30 and 150 degrees, respectively. Higher discomfort in darker red, lower in lighter blue.

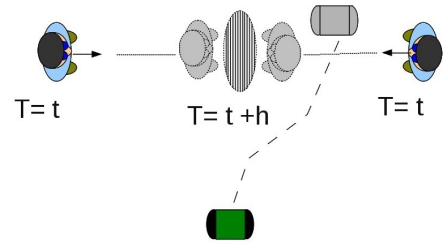


Fig. 3. An example of prediction: the robot anticipates humans' movements and avoids them.

When two people are interacting the o-space is created by the intersection of the two IPS, as we can see in the case presented in Fig. 2 (c). Finally, we use a Gaussian function centered on the pedestrian position to represent the Personal Space; the front is wider than the back as presented in [2]. Using these equations we can get the next graphics for the models: the first one is the Personal Space for a pedestrian walking in the direction of y-axis, the second one the IPS for the same case and the third one shows the resulting o-space for two pedestrians in conversation. The robot must avoid the red regions while navigating.

B. Movement Prediction

As already stated, our intent is to consider a dynamic environment where the people $\{p^{(i)}\}$ are moving. The objective function is then time-dependent and in general it will be different for each time step:

$$J_t = J(x^{(R)}; \{p_t^{(i)}\}) \,. \tag{12}$$

In this case, in order to solve the optimization problem, i.e. finding the optimal next robot position, the result can be considerably improved if we consider the function J_{t+1} instead of J_t, where:

$$J_{t+1} = J(x^{(R)}; \{p_{t+1}^{(i)}\}) \,. \tag{13}$$

This function is obviously unknown at time t but it could be approximated if a prediction model is available. Indeed, we can express the positions $\{p_{t+1}^{(i)}\}$ by means of a limited set of q past configurations

$$\{p_{t+1}^{(i)}\} = g(\{p_t^{(i)}\}, ..., \{p_{t-q}^{(i)}\}) \tag{14}$$

where the new function g represents the prediction model. In our case we do not assume any particular model and the function g is to consider completely unknown. Hence also the function

$$J_{t+1} = J(x^{(R)}; g(\{p_t^{(i)}\}, ..., \{p_{t-q}^{(i)}\})) \tag{15}$$

is now unknown. The strategy to approach the problem is not to explicitly predict the humans' movement but try to directly approximate the cost function (15) using its available past values. To do this in practice, we construct at each time step an approximator $\hat{J}_t$, like in (4), of the unknown function J_{t+1} using the last $m > q$ numerical values of J_t such that:

$$\hat{J}_t(x_t^{(R)}; \{p_{t-1}^{(i)}\}, ..., \{p_{t-q-1}^{(i)}\}) \approx J(x_t^{(R)}; \{p_t^{(i)}\}) \,. \tag{16}$$

In this way, using the last available set of humans' positions, we have an indirect approximation of the humans' movement prediction and we obtain

$$\hat{J}_t(x^{(R)};\{p_t^{(i)}\},...,\{p_{t-q}^{(i)}\}) \approx J_{t+1} \tag{17}$$

i.e., the function we want to optimize.

Once the optimization problem is defined, a fundamental point for a good behavior of the algorithm is an appropriate choice of the form of the regressor vector ϕ, introduced in equation (4). Several different choices for its explicit expression are admissible and, for the particular application treated in this paper, it was found that it suffices to choose the regressor vector as follows:

1) choose the size of the function approximator L to be an odd number;
2) select the first term of the regressor vector ϕ to be the constant term;
3) select randomly the next $(L-1)/2$ terms of ϕ to be any 2nd-order terms of the form $x_a^{(i)} \cdot x_b^{(j)}$ [with $a, b \in \{1, \ldots, \dim(x^{(i)})\}, i, j \in \{1, \ldots, M\}$ randomly-selected positive integers];
4) select the last $(L-1)/2$ terms of ϕ to be any 3rd-order terms of the form $x_a^{(i)} \cdot x_b^{(k)} \cdot x_c^{(j)}$ [with $a, b, c \in \{1, \ldots, \dim(x^{(i)})\}, i, k, j \in \{1, \ldots, M\}$ randomly-selected positive integers].

After the setting of the regressor vector ϕ and once the values of the cost function are available for measurement, it is possible to find at each time step the vector of parameter estimates θ_k and thus the approximation of the cost function $\hat{J}_k$. Then, another important choice in order to assure the convergence of the algorithm is the expression of α_k, defined in equation (6). A typical choice for such a sequence is given by

$$\alpha_k = \frac{\gamma}{(k+1)^\eta}, \tag{18}$$

where γ is a positive user-defined constant and $\eta \in (0, 0.5)$.

Remark 3: Please note that the CAO algorithm's computational requirements are dominated by the requirement for solving the least-squares problem (5). As the number of free parameters in this optimization problem is L, most popular algorithms for solving least-squares problems have, in the worst case, $\mathcal{O}(L^3)$ complexity. $\diamond$

IV. Performance Evaluation

In this section several scenarios are presented to show the execution of our algorithm in simulation. The first scenario is shown in Fig. 4: in this case five humans are present, three of them are moving and two interacting. The robot starts at (1,1) and reaches its goal while avoiding people and o-space of interaction. In Fig. 5 four different and more complex scenarios are presented. In (a) a robot has to pass through a corridor while two humans are chatting in the middle. It is possible to see how the robot is able to understand the interaction and to avoid them without disturbing. We can notice how the method evaluates many points that fall in the shortest path but finally can found a more comfortable way. In Fig. 5(b), the robot start position is aligned with the goal position but as one people is looking to the walls the chosen path guides the robot toward the middle of the corridor and then to the goal. We can remark that in this case, since the two people are not interacting, the robot can pass between them without trouble. A representation of a room with people inside is exhibited in Fig. 5(c). Here the chosen path does not interrupt any human. Last example is shown in Fig. 5(d), where the robot respects o-space of the group and p-space of humans. Note that in every simulation the presence of obstacles does not create any problem to the robot navigation. Additionally, the proposed algorithm, due to the random generation of next state configuration, is able to overcome many of typical local minima generated by obstacle avoidance problems.

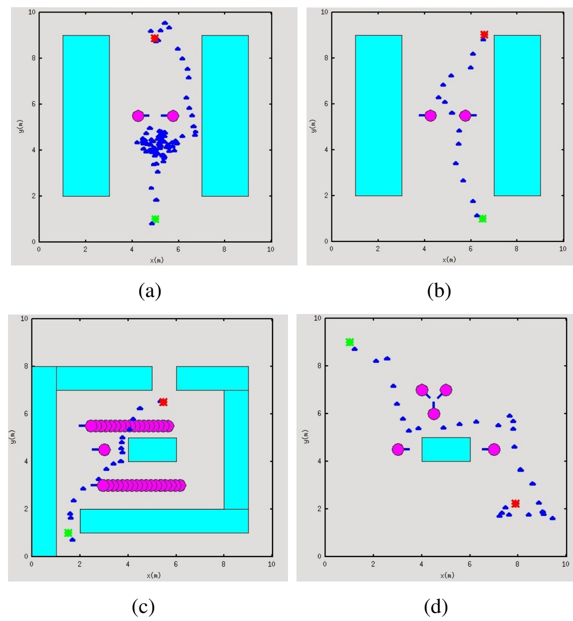


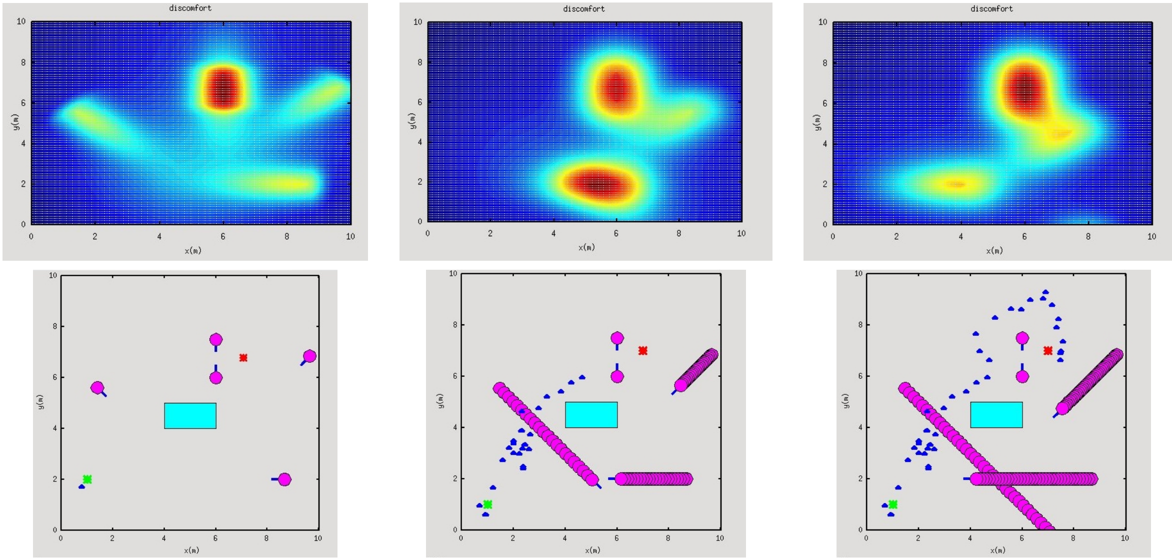
Fig. 5. More simulations with different scenarios. Start positions are in green, goal positions in red. In (a) the robot decides to take a path that minimizes discomfort of interacting humans. In (b) a similar configuration but humans are not interacting. In (c) and (d), two different complex scenarios where the robot's trajectories respect people comfort.

A. Experimental platform

The current approach is being implemented in our experimental platform, an automated wheelchair (Fig. 6(a)) equipped with two Sick lasers and a Microsoft Kinect, running ROS (Robotic Operating System) for achieving semi-autonomously mobility actions commanded by the wheelchair's user. Laser permits us to build a map of the environment, like shown on the bottom of Fig. 6(b). Data coming from the Kinect will allow us to have position and orientation of pedestrians in the scene.

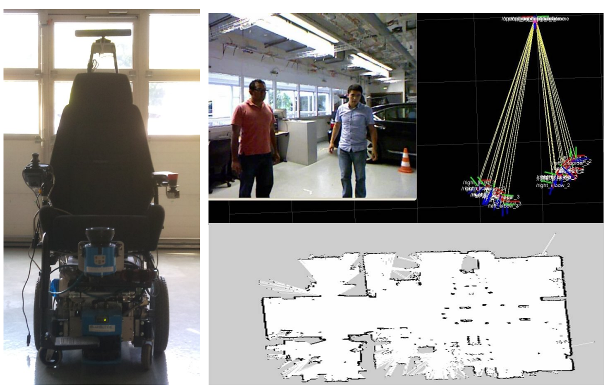
V. Conclusions and Future Work

In this paper we have presented a new stochastic optimization algorithm to move a robot in a complex, dynamic and unknown environment taking into account the respect



(a) (b) (c)

Fig. 4. Simulation of the robot navigating in an environment populated by people at three different times. Three humans walking and two in conversation. The discomfort function is shown on the top. People are represented by circles, robot's positions by small triangles, in green and red initial and goal position respectively.



(a) (b)

Fig. 6. Experimental platform: in (a) the wheelchair, on the top of (b) the data provided by the kinect, on the bottom the final map.

of humans' comfort. In particular, the proposed approach presents the following advantages:

- It does not require any a priori map of the environment
- It can include a prediction of the humans' movement
- It can easily incorporate any kind of dynamical and environmental constraints
- The random next-state searching allows us to overcome many local minima
- Low computational complexity, allowing real time implementations

The results obtained in this work are a strong motivation to continue the research and to implement the method in a real dynamic environment using the wheelchair previously described.

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