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Planning Sub-Optimal and Continuous-Curvature Paths for Car-Like Robots

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This paper deals with path planning for car-like robot. Usual planners compute paths made of circular arcs tangentially connected by line segments, as these paths are locally optimal. The drawback of these paths is that their curvature profile is not continuous: to follow them precisely, a vehicle must stop and reorient its directing wheels at each curvature discontinuity (transition segment–circle).

To remove this limitation, a new path planning problem is proposed: two curvature constraints are added to the classical kinematic constraints taken into account. Thus, the curvature must remain continuous, and its derivative is bounded (as the car-like robot can reorient its directing wheels with a limited speed only). For this problem, the existence of solutions and the characterization of those of optimal length are shown. A method solving the forward-only problem (i.e. the problem for a car moving only forward) is then presented, and this method is compared to the classical one w.r.t. the complexity and computation time, the length of the generated paths and the quality of the tracking.

Keywords — mobile-robot, path-planning, non-holonomic-system, optimality.

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1 Introduction

This paper focuses on path planning for a car-like robot: given two positions of this robot, we search a path connecting these positions and avoiding collision with a set of obstacles. The path considers only the geometrical aspects of the movement (no time dimension), but needs to respect two classical kinematics constraints: the direction of motion must remain parallel to the main axis of the robot at each point, and the turning radius of the robot is lower bounded.

Numerous methods have been proposed to solve this problem, e.g. [14, 7, 17, 24, 22], using paths made of circular arcs of minimum radius tangentially connected by line segments. The optimality (in length) of these paths has been proved by Dubins in the forward-only case [5], and by Reeds and Shepp for a robot doing backup-manoeuvres [19]. The drawback of these paths is the discontinuity of their curvature profile, which makes them difficult to follow by a real robot.

To reduce this disadvantage, paths with a continuous curvature profile can be computed, these paths having polynomial coordinates [11, 18] or polynomial curvature [10, 4]. However, only a few works take simultaneously into account the obstacle avoidance, the continuous curvature profile and the kinematic constraints of a car-like robot (in fact, the problem considered is rarely clearly stated). Moreover, no comparison between these methods have been proposed, and the improvement of the tracking has never been measured.

On the contrary, this paper begins with a precise statement of the considered problem: path planning for car-like vehicle, including obstacle avoidance and classical kinematic constraints, with curvature continuity and bound of the curvature derivative. Then, the existence of solutions, of optimal solutions and their nature are discussed, using controllability demonstrations and theory of optimal control. Especially, optimal paths are proved to be made of line segments, circular arcs and pieces of clothoid[1], and will generally contain an infinity of pieces as soon as they are long enough. After these theoretical considerations, a method to solve the forward-only problem, i.e. the problem for a robot going only forward, is described. This method is equivalent to Dubins' one, and the generated paths are very similar to Dubins' paths: the curvature discontinuity in the latest paths has been replaced by pieces of clothoid. At last, the sub-optimality of the continuous-curvature paths obtained is proved, and the method is compared to Dubins' one, w.r.t. the complexity, the computation time and the improvement of the tracking. The results clearly show the interest of this new planning method.

2 Related Works

As we mentioned in the introduction, related works can be divided in three groups: the classical path planners (using paths with a discontinuous curvature profile), the paths generators (continuous curvature paths but no obstacle

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[1]A clothoid is a curve whose curvature is a linear function of the arc length.

avoidance) and theoretical studies (without any planning method).

Usual path planners, e.g. those presented in [14, 7, 17, 24, 22], consider the planning of a path respecting the two classical constraints of the movement of a car (given in the beginning of the introduction), linking two positions of the robot and avoiding collision with a set of obstacles. They generate solution paths made of circular arcs and line segments, these paths being locally optimal for the forward-only robot [5] as for the robot doing backup-manoeuvres [19]. However, these paths have a discontinuous curvature which makes them difficult to track with a real vehicle.

On another hand, paths generators computes paths with a continuous curvature profile [11, 18, 10, 4]. But few of these generators consider mobile robots with a bounded curvature, as for example [15, 23, 18], and none of them do take into account obstacle avoidance. Moreover, these works do not present any theoretical results concerning the existence of solution paths or the nature of optimal solutions.

Such theoretical results have been presented by Boissonnat, Cerezo and Leblond [1], and developed by Kostov and Kostova [12, 13], concerning a problem similar to ours, for which the curvature is continuous but unbounded and the derivative of the curvature is bounded. Boissonnat, Cerezo and Leblond showed the existence of solutions, characterized the optimal paths and proved that these paths are generally made of an infinity of pieces (and thus cannot be used). Then Kostov and Kostova presented a set of sub-optimal paths to solve this problem.

3 Statement of the Problem

In order to state the problem we consider in this work, we will present the model of our robot and the paths it can follow.

3.1 The Car-like Robot

Our robot $\mathcal{A}$ is similar to a car-like vehicle moving on a planar environment. Its body is a rectangle supported by four wheels: the two rear wheels' axle is fixed to $\mathcal{A}$'s body and the two front wheels are directional. A position of this robot is given by a configuration (x, y, θ, κ), where (x, y) are the coordinates of a reference point R of the body, θ is the orientation of the body (i.e. the angle between the x axis and the main axis of $\mathcal{A}$) and κ is the instantaneous curvature of R's curve and represents the orientation of the front wheels (cf. Fig. 1). The idea of adding the instantaneous curvature to the classical configurations comes from [1], its advantage will be described in the next section.

The robot $\mathcal{A}$ moves on a planar workspace $\mathcal{W}$, which is represented by a compact (i.e. closed and bounded) set of $\mathbb{R}^2$. This workspace is cluttered with a set of obstacles $\mathcal{B}_j$, $j \in \{1, \dots, b\}$, represented by polygonal regions. The body of $\mathcal{A}$ must avoid **contact and collision** with these regions.

The motion of $\mathcal{A}$ is also limited by two classical constraints, as the four wheels of $\mathcal{A}$ should roll without sliding. Considering the rear wheels, whose axle is fixed to $\mathcal{A}$'s body, it implies that the movement of R remains at each point parallel to the main axis of the robot, i.e.:

$$\dot{x} \sin\theta - \dot{y} \cos\theta = 0 \qquad (1)$$

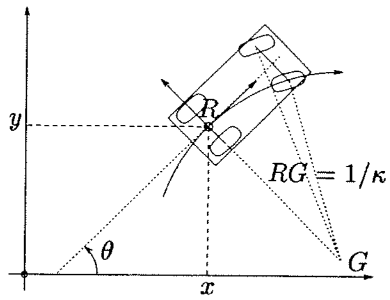

Figure 1: a car-like vehicle.

On another hand, the orientation between the directing wheels and $\mathcal{A}$'s main axis is bounded, which implies that the turning radius RG is lower bounded, or that the curvature of R's curve is bounded:

$$|\kappa| \leq \kappa_{\max} \qquad (2)$$

At last, the orientation of the directing wheels can change with a limited speed only, and thus the derivative of the curvature of R's curve remains also bounded:

$$|\dot{\kappa}| \leq \sigma_{\max} \qquad (3)$$

3.2 Feasible Paths

A path is a continuous set of positions of $\mathcal{A}$. It can be represented by a continuous curve of the configuration space $\mathcal{C} \subset \mathcal{W} \times \mathcal{S}^1 \times [-\kappa_{\max}, \kappa_{\max}]$. As a consequence, **its curvature profile** (the fourth coordinate of the configurations) **is continuous**. It is *feasible* if and only if it respects the constraints (1), (2) and (3), and is of finite length.

A feasible path can be represented by its projection on $\mathcal{W}$, i.e. by the curve R follows along this path: the orientation θ along this path is deduced using the constraint (1), its existence being ensured by the constraint (2). This representation is usually used for graphic display.

A feasible path is *smooth* if and only if its projection on $\mathcal{W}$ is C^2: along such a path, the robot moves always in the same direction (forward or backward), without back-up manoeuvres. A smooth path can also be represented by its starting configuration $q(0)$, its length l and its curvature profile $\kappa : [0, l] \longrightarrow [-\kappa_{\max}, \kappa_{\max}]$, (with $|\dot{\kappa}| \leq \sigma_{\max}$).

3.3 Planning Problem

A path Π is a mapping from $\mathbb{R}$ to $\mathcal{C}$, giving a configuration $q(s)$ for each $s \in [0, l]$, where l is the length of Π. Given a start configuration $q_s = (x_s, y_s, \theta_s, \kappa_s)$ and a goal one $q_g = (x_g, y_g, \theta_g, \kappa_g)$, such a path is a solution to our problem if and only if it links q_s to q_g and is feasible, smooth and collision-free, i.e.:

1. End conditions: $q(0) = q_s$ and $q(l) = q_g$;
2. Π is feasible and smooth, and therefore its curvature profile is a continuous function $\kappa : [0, l] \longrightarrow [-\kappa_{\max}, \kappa_{\max}]$, such that $|\dot{\kappa}| \leq \sigma_{\max}$;

3. Π is collision-free:

$$\forall j \in \{1,\ldots,b\},\ \forall s \in [0,l],\ \mathcal{A}(q(s)) \cap \mathcal{B}_j = \emptyset$$

where $\mathcal{A}(q(s))$ denotes the region of $\mathcal{W}$ occupied by $\mathcal{A}$ when in the configuration $q(s)$.

In this paper, the planners will only try to link configurations whose curvature is null. The generalization to configurations with non-null curvature is a future work.

4 Properties of the Problem

Before searching a solution to this problem, it is interesting to know whether such a solution exists. This question can be answered by proving the controllability of the robot in the considered problem. On another hand, it is also interesting to know if optimal solutions (i.e. paths with a minimum length) does exists, and to characterize these solutions.

4.1 Controllability

We have proved two results, which are equivalent to the controllability properties for the problems considered by Dubins and by Reeds and Shepp (i.e. the classical problems, without curvature continuity).

The robot is *controllable* if and only if there exists a feasible and smooth path linking any two configurations. The controllability of the robot has been proved analytically (cf. section 5.3). It means that there always exists a solution to the problem considered, as soon as there is no obstacle in the workspace.

On another hand, the robot is *small-time controllable* when it does back-up manoeuvres: for each configuration q and each neighbourhood $\mathcal{V}$ of q, there exists a second (smaller) neighbourhood of q where every configuration can be reached from q along a feasible path included in $\mathcal{V}$. It means that the kinematic constraints do not limit the existence of solutions: a feasible and collision-free path exists between two configurations if and only if a collision-free path exists between these configurations [14, Prop. 5].

The small-time controllability of the maneuvering robot can be proved using the theory of optimal control, and more precisely the Brunovsky and Lobry Theorem [16, th. IV.3]. If Π is a feasible path, its derivate (when defined, i.e. almost everywhere[2]) a function of the motion direction v (1 = forward, -1 = backward, 0 = static) and of the derivate of its curvature σ:

$$\dot{q} = F(q,\sigma,v) = vF_1(q) + \sigma F_2(q),$$

where:

$$F_1(q) = \begin{pmatrix} \cos\theta \\ \sin\theta \\ \kappa \\ 0 \end{pmatrix}, F_2(q) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

[2] A property is verified *almost everywhere* on a set if and only if there only exists a finite number of points of the set for which it is not verified.

Theorem 1 (Brunovsky and Lobry) *For $q \in \mathbb{R}^n$, we consider the system $\dot{q} = \sum_{i=1}^{p} u_i Y_i(q)$, where u_i remains in $\{-1,0,1\}$ for $i \in \{1,\ldots,p\}$. We suppose that the rank of the Lie algebra associated with $(Y_i(q), i \in \{1,\ldots,p\})$ is equal to n.*

Then, for all compact set $K \subset \mathbb{R}^n$, there exists a constant k such that, for all vector field X bounded on $\mathbb{R}^n$ by k, the system $\dot{q} = X(q) + \sum_{i=1}^{p} u_i Y_i(q)$ is controllable on the compact K.

In our case, n is 4, p is 2 and:

$$\begin{cases} u_1 &= v \\ u_2 &= \frac{\sigma}{\sigma_{\max}} \end{cases}, \begin{cases} Y_1(q) &= F_1(q) \\ Y_2(q) &= \sigma_{\max} F_2(q) \end{cases}.$$

Moreover, the Lie algebra associated with (Y_1, Y_2) is of dimension 4 [1, § 2.1] and $X = 0$ is bounded by any positive constant. Thus, our system is controllable on any compact, which implies that the associated robot is small-time controllable (with manoeuvres).

4.2 Optimal Paths

We search the optimal feasible and smooth path between two configurations q_a and q_b, without considering any obstacle. This is a Lagrange problem of optimal control [3, chapter 5], i.e. it can be stated as the optimization, for a path Π represented as a function $s \mapsto q(s)$ from $[0,l]$ to $\mathcal{C}$, of the function:

$$I[\Pi,\sigma] = g(0,q(0),l,q(l)) + \int_0^l f_0(s,q(s),\sigma(s))ds$$

with $g = 0$ and $f_0 = 1$, and the functions $q(s) = (x(s),y(s),\theta(s),\kappa(s)) \in \Pi$ and $\sigma(s)$ verifying:

- the differential system

$$\dot{q}(s) = f(s,q(s),\sigma(s)) = (\cos\theta(s), \sin\theta(s), \kappa(s), \sigma(s))$$

- the limit conditions

$$(0,q(0),l,q(l)) \in B = \{0\} \times \{q_a\} \times [0,l_{\max}] \times \{q_b\}$$

- and the constraints

$$\begin{cases} (s,q(s)) &\in A &= [0,l_{\max}] \times \mathcal{C} \\ \sigma(s) &\in U &= [-\sigma_{\max}, \sigma_{\max}] \end{cases}$$

where the upper bound $l_{\max}$ of the length of the path can be deduced of the size of the workspace $\mathcal{W}$.

We define the set $\widetilde{Q}(s,q)$, for $(s,q) \in A$, as:

$$\begin{aligned} \widetilde{Q}(s,q) &= \{(z_0,z)/\exists\sigma \in U, z_0 \geq f_0(s,q,\sigma), z = f(s,q,\sigma)\} \\ &= \{(z_0,\cos\theta,\sin\theta,\kappa,\sigma), z_0 \in [1,+\infty[, \sigma \in U\}, \end{aligned}$$

We can then apply Filippov's existence theorem for Lagrange and Bolza problems of optimal control, as stated in [3, th. 5.1.ii]:

Theorem 2 (Filippov) *If A and U are compact, B is closed, f_0, f are continuous on $M = A \times U$, g is continuous on B, and for every $(s, q) \in A$ the set $\tilde{Q}(s, q)$ is convex, then $I[\Pi, \sigma]$ has an absolute minimum as long as a solution exists.*

It is easy to verify that $\mathcal{C}$ is a compact set of $\mathbb{R}^4$, which implies that A is a compact of $\mathbb{R}^5$. Trivially, U is a compact of $\mathbb{R}$, B is a closed set of $\mathbb{R}^4$ and the functions f_0, f and g are continuous. Moreover, the set $\tilde{Q}(s, q)$ is the stripe $[1, +\infty[\times[-\sigma_{\max}, \sigma_{\max}]$ of the plane $(z_1 = \cos\theta, z_2 = \sin\theta, z_3 = \kappa)$ in $\mathbb{R}^5$, and is therefore a convex.

Thus, we know that, if a solution to our problem exists, there exists a solution whose length is optimal. To characterize the composition of the optimal solutions, we use a result obtained by Boissonnat, Cerezo and Leblond for a similar problem [1]. The only difference between their problem and ours is that their curvature is not bounded. In this condition, using Pontryagin's Maximum Principle, they have proved that the optimal paths are made of line segments and of pieces of clothoids, for which the derivative of the curvature is maximum ($\pm\sigma_{\max}$). In our case, the pieces of the optimal paths which are strictly included in $\mathcal{C}$ verify the same properties, and therefore are of the same kind. On another hand, those which are on the border of $\mathcal{C}$ corresponds (if the workspace $\mathcal{W}$ is wide enough) to circular arcs, as their curvature is constant and equal to $\pm\kappa_{\max}$. To conclude, the optimal paths of our problem are made of:

- line segments,
- pieces of clothoids of maximum derivative of the curvature,
- and circular arcs of maximum curvature.

Unfortunately, Boissonnat, Cerezo and Leblond also proved that the optimal paths of their problem are made of an infinity of pieces when they contain a line segment. Intuitively, it should imply that the optimal paths are made of an infinity of pieces, as soon as the configurations to link are far enough. The same drawback exists in our case, and therefore the optimal paths cannot be computed nor used.

4.3 Paths considered

Thus, to solve our problem, we chose to use simpler paths than the optimal ones: these path are made of at most 9 pieces, of the same kind as the pieces of the optimal paths (line segments, circular arcs and pieces of clothoids), and are called SCC-paths (for Simple Continuous-Curvature paths). These paths are very similar to Dubins' paths, but have a continuous curvature profile: the discontinuities of the Dubins' paths are replaced by pieces of clothoids (cf. Fig. 2). The sub-optimality of these paths will be proved in the results section (§ 6), after the description of the planning method.

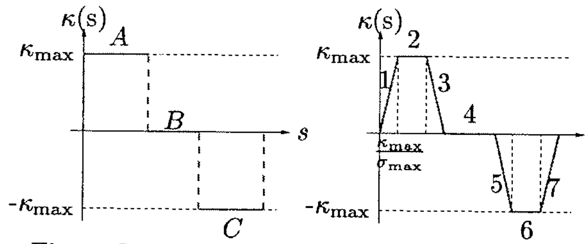

Figure 2: curvature profiles of Dubins' and SCC paths.

5 Path Planning Method

Two configurations q_s and q_g being given, we search a feasible and smooth (i.e. forward only) collision-free path linking q_s to q_g. We will describe the outline of the method, and then detail how this method is achieved.

5.1 Outline of the Method

The path planning is performed using a classical method: a fast and simple planner, called *local planner*, is associated with a higher level method to obtain the *global planner*. The local planner does not take obstacles into account, it only search the shortest (feasible and smooth) path linking two configurations, while the higher level method deals with the collision avoidance. We will mainly consider the local planner, which is our contribution to this method. A more detailed presentation can be found in last year's proceedings [21], or in the French thesis [20].

The local planner is similar to the Dubins' planner: it searches at most six paths, the circular arcs of the Dubins' paths having been replaced by *continuous-curvature turns*, made of three pieces: in Fig. 2, A is replaced by the turn made of the pieces 1, 2 and 3, and C is replaced by the turn 5–6–7. In order to apply Dubins' method, we need to find the set of the configurations that can be linked from a given configuration with a continuous-curvature turn of various deflection (i.e. variation of the orientation). It means that, for a given configuration q, we will search the set described by the final configuration of a continuous-curvature turn starting at q, when the deflection of this turn changes from 0 to 2π.

5.2 Continuous-Curvature Turns

As shown in the Fig. 2, a continuous-curvature turn is made of three pieces: a piece of clothoid of length l and of constant derivative of the curvature σ, an optional circular arc of curvature $\pm\kappa_{\max}$ and a second piece of clothoid of length l and of constant derivative of the curvature $-\sigma$.

If a continuous-curvature turn contains only two symmetric pieces of clothoid (without any circular arc), it is called *degenerated*. The non-degenerated turns verify $l = l_{\lim} = \kappa_{\max}/\sigma_{\max}$ and $\sigma = \pm\sigma_{\max}$, and correspond to a deflection greater than $\beta_{\lim} = {\kappa_{\max}}^2/\sigma_{\max}$.

Continuous-curvature turns are *symmetric* paths, i.e. their curvature profile is a symmetric curve (cf. the turns 1–2–3 and 5–6–7 in Fig. 2). As a consequence, these turns are symmetric in $\mathcal{C}$, and their projection in $\mathcal{W}$ are also symmetric (cf. [8, Prop. 1]): this is illustrated for a non-degenerated turn by Fig. 3.

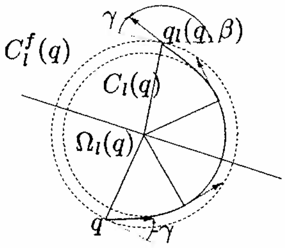

Figure 3: a continuous-curvature left turn.

All the left non-degenerated turns (for which $\sigma = \sigma_{\max}$) starting at a given configuration q contains a circular arc of the same circle $C_l(q)$ of center $\Omega_l(q)$ (cf. Fig 3). Due to their symmetry, their final configuration $q_l(q, \beta)$ remains on the same circle $C_l^f(q)$ of center $\Omega_l(q)$, for all the deflections $\beta \in [\beta_{\lim}, 2\pi]$. The radius R_T of $C_l^f(q)$ and the difference γ, between the orientation of the tangent to $C_l^f(q)$ at $q_l(q, \beta)$ and the orientation of $q_l(q, \beta)$, are two constant values depending only of the values of $\kappa_{\max}$ and $\sigma_{\max}$.

Concerning the degenerated turns, their constant derivative of the curvature σ can be chosen in $[0, \sigma_{\max}]$ for the turn to finish on the same circle $C_l^f(q)$, and with an orientation doing the same angle with the tangent to the circle, for all the deflections $\beta \in [0, \beta_{\lim}]$. The same results can be obtained for the right turns, by symmetry.

5.3 Local Planning

Once the sets of the final configuration of the continuous-curvature turns are known, the local planning method is the same as Dubins' method: at most six paths linking the given configurations are computed, and the shortest path is selected. The at most six paths are made of two continuous-curvature turns connected by a line segment or a continuous-curvature turn. If the line segment is noted s, and the turns are noted l or r when it correspond respectively to a left or a right turn, the considered paths are noted lsl, lsr, rsr, rsl, rlr and lrl. If there always exists at least four paths in Dubins' case, we have proved that there exists at least two paths in our case. It proves thus that the forward-only robot is controllable in our case.

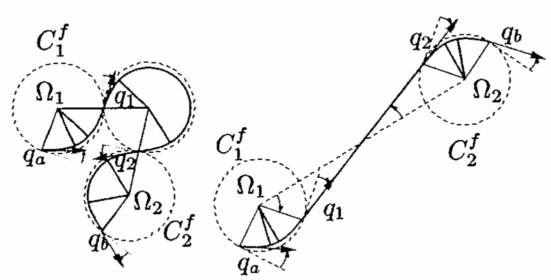

Figure 4: two examples of local planning.

Fig. 4 shows two examples of local planning, the first one corresponding to a shortest SCC-path of type lrl, the second being of type lsr.

5.4 Global Planning

Having defined a local planner, a collision checking method and a high level method are needed in order to achieve global planning.

The region swept by the robot while following a SCC-path is evaluated accurately along the line segment and circular arcs: this region is a generalized polygon, i.e. a polygon whose edges are line segments or circular arcs. This region is hierarchically approximated using motion polygons (cf. Fig. 5) along the clothoid pieces: the region cannot be computed exactly as clothoids are not analytic curves, hierarchical approximation is the most efficient representation.

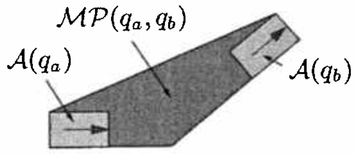

Figure 5: the motion polygon.

The high level method used to obtain a global planner is the Probabilistic Path Planner introduced by Šestka and Overmars [24].

6 Results of this Method

In this section, we will mainly compare Dubins' local planning with ours, w.r.t. the complexity, the length of the paths generated and the quality of the tracking. We will also present some experimental results obtained with the global planner we implemented.

6.1 Complexity of the Computation

Dubins' and our local planner have the same algorithm, the formulas in our case being a little more complex: Dubins' case correspond to our case, when $\sigma_{\max} \to \infty$ (or $\gamma = 0$ and $R_T = 1/\kappa_{\max}$). They have therefore the same complexity, and equivalent computing time: the computation of a SCC-path is between 1.5 to 2 times longer than the computation of a Dubins' path (cf. Table 1, presenting the results of a million tests).

Time (ms)	Minimum	Average	Maximum
Dubins' Paths	1.1	1.83	2.2
SCC-Paths	2.3	3.05	3.5

Table 1: computation time of the local planners.

Including collision checking to local planning increases similarly the time needed in both case: the computation time ratio is the same with or without collision checking (cf. Table 2).

But, if the standard deviation is near 0.06 for both planners without collision checking, it is increased with collision

Time (ms)	Minimum	Average	Maximum
Dubins' Paths	7.5–8	8.5–9	10–11
SCC-Paths	9–9.5	10.5–11	16–17

Table 2: computation time including collision test.

checking to 0.35 in Dubins' case, 0.75 in ours. This represents the fact that collision checking takes a wide range of computation time: collision (or non-collision) can be easily detected as it can be very hard to point out.

6.2 Length of the Paths

First of all, let us demonstrate the sub-optimality of the SCC-paths, with a large bound: the length of a continuous-curvature turn of deflection β is less than $\beta/\kappa_{\max} + \kappa_{\max}/\sigma_{\max}$; when the SCC-path contains a line segment, the length of this segment is less than the distance between the two circles it connects: cf. the second example of Fig. 4, $q_1q_2 \leq \Omega_1\Omega_2$ because the angle $\Omega_1 q_1 q_2$ is more than $\pi/2$. The distance $\Omega_1\Omega_2$ between the two circles is itself less than the distance d between the positions of the two configurations q_a and q_b, plus twice the radius R_T of the circles. As a conclusion, the length of the SCC-paths is always less than $d + 2R_T + 6\pi/\kappa_{\max} + 3\kappa_{\max}/\sigma_{\max}$, where d is the distance between the positions of the two configurations to connect, which is smaller than the distance between the two configurations, which is itself smaller than the length of the optimal path.

Experimental comparison between the length of Dubins' paths and the length of the SCC-paths intuitively shows that a smaller bound than this one (i.e. $2R_T + 6\pi/\kappa_{\max} + 3\kappa_{\max}/\sigma_{\max}$) may surely be found. However, it would need complex computation, and has not yet been determined.

	Minimum	Average	Maximum
Ratio	1	1.077	8.27

Table 3: ratio of lengths.

The experimental comparison are sum up in Table 3. Moreover, 82% of the SCC-paths are less than 10% longer than the corresponding Dubins' path, and thus the standard deviation of the length ratio is less than 0.2.

6.3 Quality of the Tracking

Examples of tracking have been simulated, using a Kanayama's law as described in [9]. The maximum distance between the planned path and the followed path are given in Table 4, for two different speeds (1–3 m/s). In the "wide turns" paths, the turns (circular arcs in Dubins' case, or continuous-curvature turns in our case) are followed by long line segments: in Dubins' case, the control method can come back to the planned path before arriving to the next turn. In the "zigzags" paths, the turns are adjacents: the tracking errors of the turns add one to the other. With a velocity of 3 meter per second, the control method comes to a blocking situation with Dubins' zigzags. In our case, the tracking errors always remain reasonable, and are equivalent for both types of paths.

The SCC-path planner has been used with a reactive fuzzy controller [6], to control one of the experimental vehicle (a Ligier, cf. 6) of the SHARP project.

Distance (cm)	Dubins' Path	SCC-Path
Wide Turns	35–180	<1–11
Zigzags	150–∞	<1–16

Table 4: distance between followed and planned path.

Figure 6: the experimental vehicle.

6.4 Examples of Global Planning

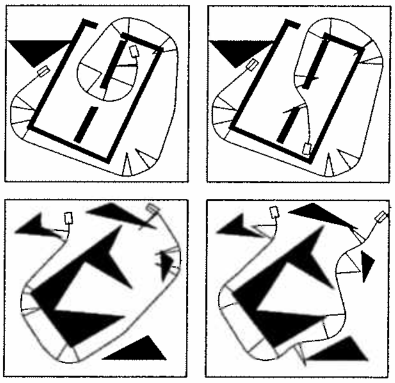

Figure 7: global planning.

Fig. 7 shows a few examples of global planning, where the rays delimit the circular arcs of the SCC-paths. These results are rather similar to those obtained with Dubins' paths, the major advantage being the continuity of the curvature (discussed in the previous sections).

7 Conclusion and Future Works

This paper describes a new path planning problem, adding two curvature constraints (continuity and bound on its derivative) to the classical kinematic constraints of a car-like robot. The new results are the characterization of the problem, w.r.t. the existence of solution and the optimality of these solutions, and experimental comparison between Dubins' local planner (usually used) and the continuous-curvature local planner. The complexity of this one is equivalent to the complexity of Dubins' one, and its computation time is less than twice Dubins' one. On another hand, Dubins' paths are more than ten times harder to track than continuous-curvature paths. The continuous-curvature local planner is thus more efficient than Dubins' one.

Future works will explore two main directions:

1. improvement of the forward-only planning; this includes the development of local and global planning

between configurations with non-null curvature (the global planning will then be complete), and the determination of the type of the shortest path (this is a generalization of the work of Bui et al. presented in [2]), so that local planning will not require to compute at most six paths;

2. study of the case with manoeuvres; the controllability in this case has been proved in this paper, as well as the nature of the optimal paths, but the positions along these optimal paths corresponding to changes of direction of motion still need to be point out; the planning of continuous-curvature paths with manoeuvres will there be a simple consequence of the previous results.

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