

Detecting guaranteed collision-free robot trajectories in unknown and unpredictable environments

Rayomand Vatcha and Jing Xiao

Abstract—For a robot to operate in a completely unknown environment, where obstacles are unknown and whether and how they move are also unknown, motion planning is largely an open problem. One essential challenge is how to guarantee that a robot can safely navigate in such an environment. We introduce a general approach to detect in real-time, based on sensing, if a future robot trajectory, which is a curve in the unknown configuration-time space, will be guaranteed continuously collision-free or not, no matter how obstacles move. Our detection algorithm efficiently uses low-level sensor data directly. Our approach does not need to identify obstacles or assume any obstacle geometry, and as such, does not base detection on predicting obstacle movements or assuming possible ways of obstacle movements. It can be used by real-time motion planners for any robot, including mobile robots and manipulators, to guide the robots motion in unknown and unpredictable environments. As long as a robot is moving along a detected collision-free trajectory by our approach, its safety is guaranteed, i.e., it will not be hit by any obstacle.

I. INTRODUCTION

Motion planning for a robot moving in an uncertain, dynamic environment is gaining more attention in the robotics research community. One common assumption about the available information for such planning is known obstacle geometry. Another assumption is certain knowledge of obstacle motion [1] [2]. If the motion of an obstacle is unknown, a common approach is to predict the future motion by tracking the past obstacle motion (e.g. [3]–[8]). There are motion planners based on prediction for mobile robot motion planning (e.g., [9]–[11]) and for mobile manipulator motion planning [12].

However, prediction can only be sufficiently accurate for a short period, i.e., immediately after the time when the prediction is made. To compensate for that requires frequently repeated prediction and computation for collision-checking. Moreover, the planned robot motion is still not guaranteed collision-free due to the possibility of wrong predictions.

There is less research addressing guaranteed collision-free motions. The notion of “Inevitable Collision Regions” (ICS) was introduced [13] for a mobile robot to characterize guaranteed CT-obstacles in its CT-space. In [14], the motion planner avoided prediction of obstacle motion by considering all the possible obstacle motions so that the planned motions

are guaranteed collision-free, however, at the expense of assuming a much smaller free space for robot motion than the actual free space in the robot’s configuration-time (CT) space.

Most motion planning approaches assume the knowledge of obstacle geometry or recognizable obstacles. However, it is far from trivial to acquire such knowledge via sensing in an unknown and dynamic environment. There is good progress in detecting and recognizing obstacles in some city road settings or off-road settings (e.g. [15], [16]), such as vegetation (e.g., [17]), people (e.g., [18]), etc. However, in very crowded and dynamic environments with many unknown changes, such as human-centered environments, recognizing all obstacles can be too challenging and also unnecessary. Vast number of objects of different kinds can appear or disappear, become separate into smaller objects or combine into bigger ones. Thus, it is desirable to study how to enable motion planning for robots without the need of recognizing unknown obstacles that may also move in unknown ways.

We have introduced a general approach [19] to detect at sensing time τ whether a robot (which can be of high-DOF, such as a manipulator) will be *guaranteed* collision-free at configuration-time (CT) point (\mathbf{q}, t) ($t > \tau$) without requiring segmentation or recognition of obstacles and predictions of their motions. The approach is based on two unique concepts: *atomic obstacles* and *dynamic envelope*. We have demonstrated that detection can be efficiently performed [20]. We have further shown that [21] a guaranteed continuously collision-free trajectory can be detected by checking whether a special set of discrete CT points are guaranteed collision-free. This paper summarizes these results.

II. ASSUMPTION AND NOTATIONS

It is reasonable to assume that even in an unknown and unpredictable environment, all obstacle speeds are bounded to be below a certain maximum possible speed v_{max} , which can be an over-estimated upper-bound. For example, the speed of fastest vehicle, fastest human runner, etc. are known. Of course, an obstacle may have varied actual speeds in $[0, v_{max})$. Throughout this paper, we simply consider any obstacle’s speed to be on $[0, v_{max}]$.

The following notations describe a robot model in the Cartesian space (i.e., the physical space).

- $R(\mathbf{q})$: the region occupied by a robot R at configuration \mathbf{q} .

This research was supported by the US National Science Foundation under Grant IIS-0742610.

R. Vatcha is a PhD student in Computer Science, University of North Carolina at Charlotte, USA. rvatcha@uncc.edu

J. Xiao is with the faculty of Computer Science, University of North Carolina at Charlotte, USA. xiao@uncc.edu

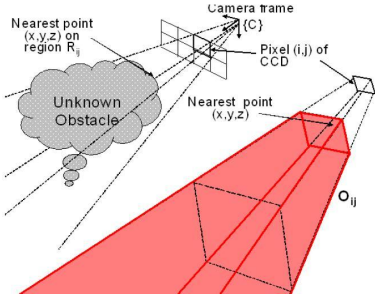


Fig. 1. The geometry of an atomic obstacle O_{ij} that originates from low-level sensor data (x, y, z) , generated by a stereo vision sensor.

We also use the following different temporal notations in the description of a robot's operation:

- τ : time of sensing
- t : time of action

III. ATOMIC OBSTACLES: REPRESENTING OBSTACLES AT A SENSING INSTANT

We introduce the concept of *atomic obstacles* that directly uses low-level sensor data to represent real obstacles at any sensing instant in an unknown, changing environment without elaborate sensor information processing. Without the loss of generality, the lower-level sensory data, generated by whatever sensor (e.g., laser range finders, sonar, stereo vision, etc.), can be treated as atomic obstacles of similar and simple geometry at different locations. Collectively, the atomic obstacles represent actual obstacles in an environment for just one sensing moment, and we do not relate the atomic obstacles of one sensing moment to those of the next.

As a concrete example, consider an overhead stereo-vision sensor that provides an image of an unknown environment. Every pixel (i, j) of that image maps to a surface region W_{ij} of 3-D points in the physical world. The sensor generates the 3-D point (x, y, z) in W_{ij} that is closest to the image plane.

Since W_{ij} occludes the space behind it, to be safe, W_{ij} and the infinite volume of points it occludes can be viewed as an atomic obstacle O_{ij} that a robot cannot collide with at that sensing moment. O_{ij} , associated with a pixel (i, j) of the image, starting from the point (x, y, z) , and extending towards infinity, can be viewed as a trapezoidal ray originating from (x, y, z) as shown in the Figure 1. The environment can now be viewed as consisting of *only* these atomic obstacles O_{ij} for all (i, j) 's in the image; see Figure 2 for an example.

It does not matter that the atomic obstacles as defined above change with the viewing direction, because we are not concerned with what the actual obstacles look like or how to relate them from one image to the next. Thus, the low-level sensory data obtained at a sensing instant is only useful until the next sensory data are obtained, and hence, should be replaced entirely by the next sensory data. In other words, there is no need for accumulating sensory data, and the space complexity for storing sensory data is simply a

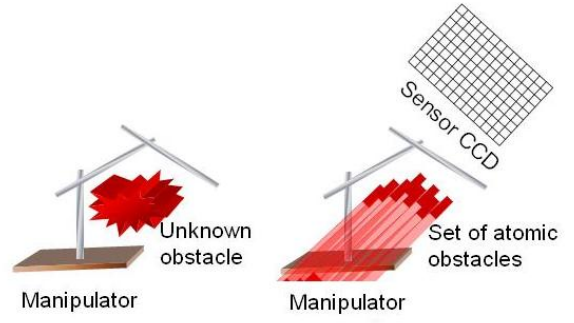


Fig. 2. An environment is viewed as a set of atomic obstacles at a sensing instant. Reprint from [19].

constant. For example, for the stereo-vision sensor the space complexity is the size of the image.

The atomic obstacles also need not be of the same size. However, it is important that the atomic obstacles come directly from sensory data and are of simple shapes.

IV. DYNAMIC ENVELOPE: DETECTING GUARANTEED COLLISION-FREE CT-POINTS NO MATTER HOW OBSTACLES MOVE

At some sensing instant τ_0 , we aim to detect if a future CT-point $\chi = (\mathbf{q}, t)$, with $t > \tau_0$, is *guaranteed* collision-free. $\chi = (\mathbf{q}, t)$ is not collision-free only if the following *worst-case scenario* will happen during $[\tau_0, t]$: the nearest obstacle to $R(\mathbf{q})$ at τ_0 will move towards $R(\mathbf{q})$ with maximum speed v_{max} and collide with $R(\mathbf{q})$ at $t_i \in [\tau_0, t]$ and then it will stop there to keep the collision. Since many other scenarios are possible, one clearly should not, at time τ_0 , treat χ as not collision-free by assuming the above worst-case scenario.

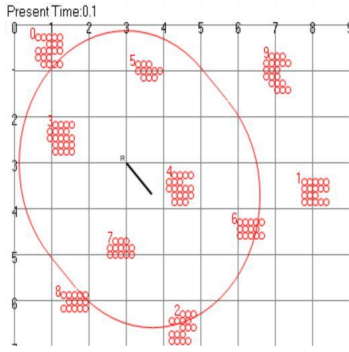
Instead, one should keep observing during the time interval $[\tau_0, t)$ how obstacles perform w.r.t. $R(\mathbf{q})$ and discover if χ is actually collision-free or not. This is the insight behind the novel concept of *dynamic envelope*, which discovers collision-free CT points by capturing actual scenarios *without* assuming the worst-case scenario or any particular kinds of obstacle motions.

Definition 1: For a CT-point $\chi = (\mathbf{q}, t)$, a *dynamic envelope* $E(\chi, \tau_i)$, as a function of current sensing/time $\tau_i \leq t$, is a closed surface enclosing the region $R(\mathbf{q})$ such that the minimum distance between any point on $E(\chi, \tau_i)$ and $R(\mathbf{q})$ is

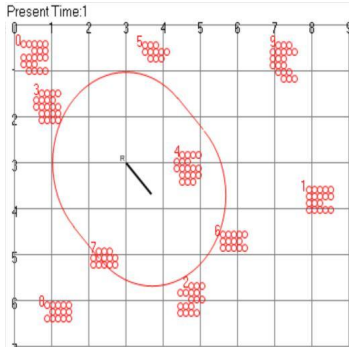
$$d_i = v_{max}(t - \tau_i) \quad (1)$$

The following are major properties of a dynamic envelope $E(\chi, \tau_i)$, which capture non-worst case future obstacle motions, without assuming any particular kinds of obstacle motion:

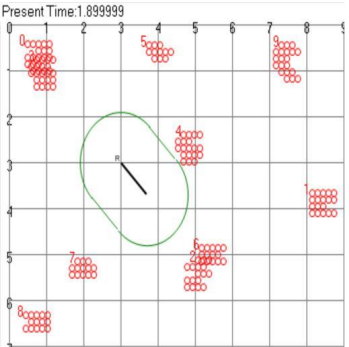
- 1) A dynamic envelope shrinks monotonically over sensing time with speed v_{max} , i.e., $E(\chi, \tau_i) \supset E(\chi, \tau_{i+l})$, where $l > 0$, $\tau_i < \tau_{i+l} \leq t$.
- 2) An obstacle outside $E(\chi, \tau_i)$ will never intersect $E(\chi, \tau_{i+l})$, since an obstacle cannot move faster than v_{max} .



(a) As $E(\chi, 0.1)$ contains atomic obstacles, it is uncertain if real obstacles 2-8 will collide with the robot at χ .



(b) At $\tau_i = 1$ s, the real obstacle 6 is just “squeezed out” of $E(\chi, 1)$, i.e., obstacle 6 did not conduct worst-case movement.



(c) $E(\chi, 1.89)$ contains no atomic obstacle, meaning that no real obstacles 2-8 conducted worst-case movements.

Fig. 3. Dynamic envelope of a planar rod robot at $\chi = (\mathbf{q}, t) = ((3, 3), 3)$. The atomic obstacles are red circles. Their clusters represent real obstacles. Reprint from [19].

- 3) An obstacle intersecting $E(\chi, \tau_i)$ can be “squeezed” out of $E(\chi, \tau_{i+t})$, for certain τ_{i+t} , if *not* moving towards $R(\mathbf{q})$ in the maximum speed v_{max} .

Thus, at sensing time $\tau_i < t$, if the dynamic envelope $E(\chi, \tau_i)$ is detected free of atomic obstacles, it is also free of actual obstacles, and the robot will surely be collision-free at $\chi = (\mathbf{q}, t)$; else the robot may or may not be collision-free at $\chi = (\mathbf{q}, t)$ (i.e., it is uncertain).

Figure 3 shows an example. $\chi = ((3, 3), 3)$ and $v_{max} = 1$ unit/s. At $\tau_i = 1.89$ s, χ is perceived collision free.

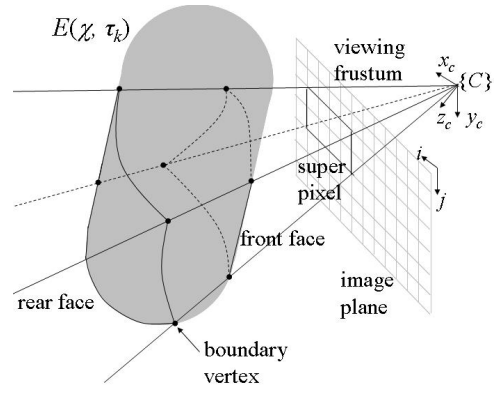


Fig. 4. Grouping atomic obstacles for faster collision checking, Reprint from [20].

V. COLLISION DETECTION IN REAL-TIME

The concept of dynamic envelope, coupled with atomic obstacles, enables the detection of collision-free CT-points regardless of how obstacles look like and how they move in an unknown and changing environment. By observing a shrinking dynamic envelope over time and performing intersection checking between a dynamic envelope and atomic obstacles at each sensing moment, one can catch the earliest sensing moment when the dynamic envelope does not include atomic obstacles, i.e. a (future) CT-point is detected guaranteed collision-free.

As each atomic obstacle corresponds to a pixel of an image from sensing, such as a stereo vision image, depending on the image resolution, there can be a great number of atomic obstacles. For example, even an image with a coarse resolution of 188×120 generates up to 22,560 atomic obstacles. Thus, key to real-time efficiency for checking if a CT is guaranteed collision-free is how to manage the large number of atomic obstacles. To minimize the number of intersection computations between dynamic envelope and atomic obstacles, the following strategies are introduced [20]:

- **Extraction:** Consider only those atomic obstacles that are likely to intersect with a dynamic envelope, i.e., the atomic obstacles whose indices (i, j) are on the projection $P(E)$ of the dynamic envelope on the image plane.
- **Grouping:** Partition pixels on $P(E)$ into multi-size *super pixels*, such that each super pixel corresponds to a $m \times n$ image region of $P(E)$, with varied $m (\geq 1)$ and $n (\geq 1)$ values¹. The atomic obstacles corresponding to a super pixel on $P(E)$ form a *combined atomic obstacle*. With such grouping, intersection checking is reduced to that between the dynamic envelope and combined atomic obstacles (which are far fewer than the atomic obstacles). Figure 4 shows such grouping of atomic obstacles.
- **Hierarchical Checking:** Perform intersection checks efficiently through multi-level simplified computations by subdividing a combined atomic obstacle into smaller

¹When $m = n = 1$, the super pixel is reduced to a normal pixel.

ones when an intersection is detected between a dynamic envelope and that combined atomic obstacle. Thus, if no intersection is detected at a high-level, then there is no intersection for sure; else, re-check intersection at a lower level.

The collision detection algorithm achieves real-time efficiency, as confirmed by the experimental results in [20].

VI. DETECTING CONTINUOUSLY COLLISION-FREE TRAJECTORIES

In [21], we have further shown that if a CT point (\mathbf{q}, t) is discovered collision-free, a neighborhood (CT-region) of (\mathbf{q}, t) is also guaranteed collision-free. Based on that, given a continuous robot trajectory, we have presented a method to compute a set of discrete CT-points such that, if these points are discovered to be guaranteed collision-free, their associated collision-free neighborhood CT-regions contains the continuous trajectory, i.e., the trajectory is guaranteed continuously collision-free. Note that prior approaches in the literature for detecting continuously collision-free paths/trajectories require either knowing future motion of the obstacles or that the swept volume traversed by a robot along a path/trajectory can be computed; such approaches cannot be used here in real-time.

Given a CT point $\chi = (\mathbf{q}, t)$ and the sensing instant τ_i , the dynamic envelope $E(\chi, \tau_i)$ represents the expansion of the region $R\mathbf{q}$ in the physical space in all directions by the distance $v_{max}(t - \tau_i)$. Thus, there exists a continuous neighborhood C_q of configurations that the robot can move to without being outside of $E(\chi, \tau_i)$ in the physical space, and there are corresponding smaller dynamic envelopes inside $E(\chi, \tau_i)$, see Figure 5, each corresponds to a CT point $\chi' = (\mathbf{q}', t')$, such that $\mathbf{q}' \in C_q$, and $\tau_i \leq t' < t$. Hence, there is a continuous CT region $F(\chi, \tau_i)$ corresponding to $\chi = (\mathbf{q}, t)$ formed by those χ' s.

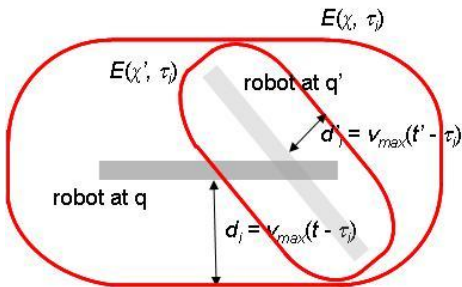


Fig. 5. A dynamic envelope $E(\chi', \tau_i)$ inside the dynamic envelope $E(\chi, \tau_i)$ for a planar rod robot

If $E(\chi, \tau_i)$ is free of atomic obstacles so that the CT point $\chi = (\mathbf{q}, t)$ is guaranteed collision-free, then all CT points in the CT region $F(\chi, \tau_i)$ are also guaranteed collision-free.

Given a trajectory Γ in the CT space, we then find a set $Q(\Gamma)$ of discrete CT-points such that when those CT points are detected as guaranteed collision-free, the union of their corresponding CT-regions, which is also detected collision-free, covers the trajectory Γ . In other words, through detecting if the CT points in set $Q(\Gamma)$ are collision-free, we

can detect if the trajectory Γ is continuously collision-free. An implemented example in [21] demonstrated the real-time usage of this method.

VII. CONCLUSIONS

We have introduced a general approach to detect in real-time, based on sensing, if a future robot trajectory, which is a curve in the unknown configuration-time space, will be guaranteed continuously collision-free or not, no matter how obstacles move. Our approach can be used by any real-time planner for detecting safe robot trajectories. As future research, we will investigate how to incorporate moving sensors in this approach and systematically take into account sensing uncertainty. We will also investigate how to relate sensing frequency with computational cost to maximize efficiency for collision checking.

While the concepts of atomic obstacles and dynamic envelope are introduced together, they do not have to be used together. If the geometry of an obstacle is known and sensing can provide the information of an obstacle's pose, intersection checking can be done directly between a dynamic envelope and such an obstacle to detect guaranteed collision-free robot trajectories. Prediction of obstacle motion is avoided.

REFERENCES

- [1] P. Fiorini and Z. Shiller, "Motion planning in dynamic environments using velocity obstacles," in *Intl. J. of Robotics Research*, pp. 760–772, 1998.
- [2] F. Large, S. Sckhvat, Z. Shiller, and C. Laugier, "Using non-linear velocity obstacles to plan motions in a dynamic environment," in *IEEE Intl. Conf. on Control, Automation, Robotics and Vision (ICARCV)*, pp. 734–739, 2002.
- [3] A. Ess, B. Leibe, K. Schindler, and L. V. Gool, "Moving obstacle detection in highly dynamic scenes," in *IEEE Intl. Conf. on Robotics and Automation*, pp. 56–63, May 2009.
- [4] G. Gallagher, S. S. Srinivasa, J. A. Bagnell, and D. Ferguson, "Gatmo: a generalized approach to tracking movable objects," in *IEEE Intl. Conf. on Robotics and Automation*, pp. 2043–2048, May 2009.
- [5] C. C. Chang and K.-T. Song, "Environment prediction for a mobile robot in a dynamic environment," *IEEE Trans. on Robotics and Automation*, vol. 13, pp. 862–872, Dec 1997.
- [6] A. Elnagar and K. Gupta, "Motion prediction of moving objects based on autoregressive model," *IEEE Trans. on Systems, Man, and Cybernetics (Systems and Humans)*, vol. 28, pp. 803–810, November 1998.
- [7] A. Elnagar and A. Hussein, "An adaptive motion prediction model for trajectory planner systems," in *Intl. Conf. on Robotics and Automation*, pp. 2442–2447, September 2003.
- [8] V. Govea, D. Alejandro, F. Large, T. Fraichard, and C. Laugier, "Moving obstacles' motion prediction for autonomous navigation," in *Int. Conf. on Control, Automation, Robotics and Vision*, December 2004.
- [9] V. Govea, D. Alejandro, F. Large, T. Fraichard, and C. Laugier, "High-speed autonomous navigation with motion prediction for unknown moving obstacles," in *IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, pp. 82–87, October 2004.
- [10] A. Kushleyev and M. Likhachev, "Time-bounded lattice for efficient planning in dynamic environments," in *IEEE Intl. Conf. on Robotics and Automation*, pp. 1662–1668, May 2009.
- [11] J. van den Berg, D. Ferguson, and J. Kuffner, "Anytime path planning and replanning in dynamic environments," in *IEEE Intl. Conf. on Robotics and Automation*, pp. 2366–2371, May 2006.
- [12] J. Vannoy and J. Xiao, "Real-time Adaptive Motion Planning (RAMP) of mobile manipulators in dynamic environments with unforeseen changes," in *IEEE Trans. on Robotics*, vol. 24(5), pp. 1199–1212, Oct. 2008.

- [13] T. Fraichard and H. Asama, "Inevitable collision states - a step towards safer robots?," *Advanced Robotics*, vol. 18, no. 10, pp. 1001–1024, 2004.
- [14] J. van den Berg and M. Overmars, "Planning time-minimal safe paths amidst unpredictably moving obstacles.," in *Intl. J. on Robotics Research*, pp. 1274–1294, 2008.
- [15] A. Murarka, M. Sridharan, and B. Kuipers, "Detecting obstacles and drop-offs using stereo and motion cues for safe local motion," in *IROS*, pp. 702–708, 2008.
- [16] C. Caraffi, S. Cattani, and P. Grisleri, "Off-road path and obstacle detection using decision networks and stereo vision," *IEEE Trans. on Intelligent Transportation Systems*, vol. 8, no. 4, pp. 607–618, 2007.
- [17] D. Bradley, R. Unnikrishnan, and J. A. Bagnell, "Vegetation detection for driving in complex environments," in *IEEE Intl. Conf. on Robotics and Automation*, April 2007.
- [18] N. Bellotto and H. Hu, "Multisensor-based human detection and tracking for mobile service robots," *IEEE Trans. on Systems, Man, and Cybernetics – Part B*, vol. 39, no. 1, pp. 167–181, 2009.
- [19] R. Vatcha and J. Xiao, "Perceived CT-space for motion planning in unknown and unpredictable environments," in *8th Workshop on Algorithmic Foundations of Robotics*, (Guanajuato, Mexico), Dec 2008.
- [20] R. Vatcha and J. Xiao, "An efficient algorithm for on-line determination of collision-free configuration-time points directly from sensor data," in *IEEE Intl. Conf. on Robotics and Automation*, May 2010.
- [21] R. Vatcha and J. Xiao, "Discovering guaranteed continuously collision-free robot trajectories in an unknown and unpredictable environment," in *IEEE/RSJ Intl. Conf. on Intelligent Robots and Systems*, October 2009.